Houses across time and across place

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July 26, 2018

Abstract

This paper analyses housing costs and changing patterns of residential development over the long term in a dynamic general equilibrium. It makes four main points. First, when agglomeration matters the speed of travel improvements is crucial. We derive an intuitive condition for the rate of change in transport efficiency that generates flat land and house prices on a balanced growth path. Second, we present evidence that this condition seems to have been approximately satisfied in many developed economies between the mid-nineteenth century and the mid-twentieth century, but that since then passenger transport improvements have slowed down. Once that happens we move off the balanced growth path. Our calibrated model suggests this has been the crucial factor behind rising housing costs in many countries in recent decades. Third, we show how significant cross-country variation could be due to land availability and planning restrictions; though planning restrictions tend to have less impact. Finally, we show that speed of increase in house prices is extremely sensitive to two parameters - the elasticity of substitution between land and structure in creating housing services and substitutability between housing and consumption goods in utility. We find that in many countries with slowing improvements in transport it is plausible that house prices could now persistently rise faster than incomes.

JEL classification: R21,N90,R30,R31.

Keywords: Two Sector Growth Model, Housing Supply, Spatial Geography, House Prices.

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This paper aims to uncover the drivers of house and land prices - and of patterns of residential development - over the long term. It makes four connected points. First, when agglomeration matters, so that there are benefits to central location, the evolution of commuting costs is crucial to the determination of land and house prices. We derive a condition for the rate of change in transport efficiency that generates flat land and house prices. Along this balanced growth path, the improvements in transport efficiency make it profitable to develop less accessible land, and this increase in land supply offsets the increase in demand due to population and income growth. We present evidence that this condition seems to have been approximately satisfied in many developed economies between the mid-nineteenth century and the mid-twentieth century. However, since then passenger transport improvements have slowed down. Our second point is that by accounting for this slow down our model can account for the general path of house prices in many developed economies over the last 150 years; first nearly 75-80 years of flat land and house prices, with house prices then rising gently for another 40 years before rising more rapidly from a few decades ago. Third, we show how this general pattern for house prices is affected by either a shortage of undeveloped land or by planning restrictions. Differences in these dimensions can account for the idiosyncratic variation seen across countries in the general path for house prices. Finally, we show that when the economy is not on a balanced growth path, the evolution of home prices is greatly sensitive to two elasticities: the elasticity of substitution between land and structures in producing housing; and the elasticity of substitution between consumer goods and housing in utility. We develop a calibrated, general equilibrium model to illustrate these points and also use it to consider the path of home prices over the next one hundred years.

We use a framework that combines features of a Ramsey two-sector growth model with a model of the changing geography of residential development that tracks the change in location of the population over time. We are not aware of work which combines these features. We specify transport costs using Samuelson's (1954) 'Iceberg' cost model but apply it to commuting costs as in Krugman & Elizondo (1996); we argue, as they do, that this is a plausible model if the predominant cost to the individual is the time spent travelling. This formulation has the advantage that we are able to derive explicit functions for population density, and for land and house prices, by location and over time. It also means that our model embeds the insights of the Alonso-Mills-Muth urban model using more general functional forms than is typical for both preferences and production of housing services. Our specification of transport costs has the important additional advantage that all households have the same marginal utility of income at all locations. This allows us to calculate the dynamic consumption path of all households without making any further assumptions. Our first result is that this model supports a balanced growth path if the rate of growth of transport efficiency equals one half the rate of growth of GDP. We show evidence that this condition was broadly satisfied in many developed economies between the mid-nineteenth century and the mid-twentieth century. Thus our model provides a

micro-foundation, based on the impact of transport improvements, for the assumption of exogenous land productivity growth behind the balanced growth results in Nichols (1970).

The model illustrates how the forces driving up house and land values (growth in population and average incomes with a fixed supply of land) play out against forces making them cheaper (improvements in transport allowing people to live further from urban centers where agglomeration means the good jobs are to be found). Our focus is on how the quantity of land within commutable distance of a central location, where agglomeration benefits are greatest, is endogenously determined in a way that reflects preferences, trends in population and technology; we assess how these shape the pattern of land and home prices across time and space. We endogenise the gradient of house values with respect to distance from the center, the mix of land and structure used to create houses at different locations and the spatial pattern of population density; all these things change over time even on a balanced growth path where house prices are constant. We find that land and house prices can be constant in a growing economy even when land for residential development within any given distance of an unchanging central location is fixed. But that depends on a specific condition being satisfied. Given that technological progress of the transport and production sectors are exogenous in this model, this might appear to be a 'knife-edge' condition. However in the literature on directed technical change, originating in Kennedy (1964) and Drandakis & Phelps (1966) and recently revitalized by Acemoglu (2002), innovation R&D will be directed towards the factor in short supply if the factors are complements (an elasticity of substitution less than 1). Thus in these models with endogenous R&D effort, the equilibrium is achieved when all augmented factors grow at the same rate. For a particular historical period this condition does seem to have been approximately satisfied. However, in general the economy will not follow a balanced growth path and land and house prices will not be constant.

Over the past seventy years or so it appears that house prices in many countries have risen very substantially faster than the price of (other) goods. Knoll et al. (2017) present carefully constructed measures of average national house prices for 14 advanced economies since 1870. Their measure of national, real house prices (that is relative to consumer goods) averaged across all countries rises by about 300% in the period since 1945 (see their Figure 2). In some countries national house prices have, on average and measured over many decades, risen faster than incomes, and not just faster than consumer goods prices. In the UK house prices relative to average household incomes by 2015 were, on many measures, around double the level from the late 1970's. In the US average national house prices seem not to have risen faster than incomes over the period since the end of the second world war, though as in nearly all advanced countries they have risen substantially faster than aggregate consumer prices. Albouy et al. (2016) present evidence that housing's relative price, share of expenditure and "unaffordability" have all risen in the US since 1970. Rognlie (2016) presents evidence that the average share across G7 countries of private domestic added value that is accounted for by returns on housing has risen steadily since 1950; essentially

all of the rise in the net capital share reflects a rising share of housing.

This rise in the relative price of housing across most developed countries in the period since the second world war has come as the proportion of the population living in big cities has risen in most developed countries[1]. It has also coincided with a period where real transport costs have been flat or (more recently) often rising; that is markedly different from the period between the middle of the nineteenth century and the second world war when transport costs fell dramatically. (We review the evidence in some detail below). These phenomena - rising relative price of housing, an end to falls in transport costs, greater urbanization in population - are plausibly linked. One of the aims of this paper is to explore the nature of that link and to develop a model which accounts for many of the patterns seen in developed economies in the last 100 years or so.

We calibrate our model along the balanced growth path to an average of US macroeconomic ratios in the 1950s and 1960s; these ratios include both the value of land, and value of housing to GDP ratios as well as those that are captured by the more standard exogenous growth models. We also show if the growth in transport efficiency slows or even halts - as all the evidence suggests - then our model can reproduce the trajectory of house prices since the mid twentieth century. We are also able to consider the impact of a shortage of new land to develop as well as any building restrictions, such as zoning rules. If the urban area runs out of space to develop into then this can cause a substantial upward pressure on house prices. But we find planning restrictions that constrain the density of development tend to have less of an impact (though this is sensitive to the elasticity of substitution between structure and land in the production of housing services).

We also use the model to cast light on several issues about the evolution of house and land values over the next several decades: Can we expect housing costs (that is implied rents) and house prices to continue rising relative to the price of other goods? Why did real house prices across countries seem to rise rather little in the 75 years before around 1950 but then treble in the next 70 years? Are there conditions where house prices can be expected to persistently rise faster than incomes? What accounts for the tendency of housing costs in some countries to rise in real terms but at a rate slower than the rise in incomes while in other countries housing cost to income ratios have been on an upwards trend for decades? In addressing these issues we take agglomeration effects as a given. Our aim is not to explore the way in which city-specific agglomeration effects interact across locations to generate aggregate productivity growth, income inequality and the size distribution of cities. That is the focus of an existing and growing literature (see, for example, Davis et al. (2014), Black & Henderson (1999), Rossi-Hansberg & Wright (2007)).

We show that it is not difficult to find sets of parameters that are plausible, in the light

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[1]By 1950, only 30% of the world's population resided in cities. That share increased to 54% by 2015 and is now expected to increase to 66% by 2050. (Source: Atlas of Urban Expansion, 2016, volume 1). For developed countries these ratios are higher but have also followed an upward trajectory. Duranton & Puga (2013) note that the growth of population in large cities in developed countries has exceeded national population growth by substantial margins. Growth in population has shown up in ever larger big cities rather than in growth in the number of cities.

of existing evidence, and which imply that house prices can rise faster than incomes for periods spanning generations. But at parameter values that are close (relative to the uncertainty about empirical estimates of those parameters) house prices follow radically different trajectories. Setting key elasticities to unity - as is often done in papers assuming log utility or Cobb Douglas production of housing (see for example Kiyotaki et al. (2011), Grossmann & Steger (2016), Favilukis et al. (2017)) - turns out to be a massively important assumption. We also find that initial differences in the ratio between land area and population across countries create very different paths for housing costs over the next several decades. Differences in population density also mean that the incentives to invest in productive capital can diverge.

But one of the implications of the great sensitivity of such paths to small variations in the two key elasticities is that in a world where perfect foresight is obviously impossible - and where variability around underlying equilibrium paths might be substantial and excess exuberance might arise - it could be very hard to spot divergences in house values from sustainable levels. If a sustainable path for house prices at an elasticity of substitution between land and structures of 0.55 rises consistently faster than incomes while the path at an elasticity of 0.65 consistently falls relative to incomes then it will be difficult to figure out whether prices are diverging from a sustainable trajectory. We find that such dramatically different trajectories are indeed likely at values slightly above and slightly below what many studies find as a central estimate of key elasticities. One of the contributions of this paper is to show just how important these elasticities are and how misleading it is to draw strong conclusions from models where they are set at values that are chosen largely for analytical convenience.

Related literature

There is a large literature on the determinants of house prices, both across time and across regions within countries. Nearly all of the literature focuses either on the time dimension of average prices or on the cross section of prices by location - one notable exception is Glaeser (2008). (For a good review of the literature on this and many other aspects of housing economics see the survey paper by Piazzesi & Schneider (2016) and the many references therein). Much of the literature focuses on the variability in land and house prices, and in the density of development and populations, across regions at a point in time. The pioneering works are Alonso (1960), Alonso (1964), Mills (1967), Mills (1972) and Muth (1969); a resurgence in the literature was triggered by Krugman (1991) and Lucas (2001). We introduce variability in housing, location and land values in a tractable way that captures the essence of these ideas that there are desirable locations and that being further from them creates costs. Land and house prices adjust to reflect that. We map out how this adjustment evolves over time and in so doing model the overall macroeconomy and its path over long periods. The focus of much of the literature on national house prices is less on the very long term drivers of housing markets and more on business cycle variability in values. Much of the literature on regional differences in housing conditions

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does not focus on the macroeconomic backdrop so takes aggregate incomes, interest rates and population as given (and often constant). A substantial literature looks at how housing fits in to household decisions on portfolio allocation, borrowing and saving (see, for example, Campbell & Cocco (2003) and Campbell & Cocco (2007)). In much of this literature the changing way in which housing is supplied is not the focus of attention; supply is often assumed fixed or at least exogenous. Our focus is on the long run.

One paper in the spirit of our own is Deaton & Laroque (2001); see also Kiyotaki et al. (2011), Grossmann & Steger (2016) and Favilukis et al. (2017). Those papers, like this one, embed the housing sector within a model of the overall economy that endogenises growth, saving and asset prices. But they do not focus on the spatial dimension of housing, which we show is important when thinking about long run dynamics. Any long run analysis has to model the changing supply of housing taking into account the fixity of land mass and the way in which endogenous shifts in the cost of land relative to structures changes the way in which houses are constructed. Land is obviously not homogenous and the impact of the most important way in which it differs (that is by location) varies over time as technological change means that distance may have a varying effect on value. One obvious way in which this happens is if transport costs change.

The contrast between very rapid rises in travel speeds in the late nineteenth and early twentieth century and a more recent stagnation in speeds is stark and may be one reason why changes in real house prices seemed very much smaller in most developed countries between 1850 and the second world war than in the 70 odd years since then. In an earlier version of Knoll et al. (2017) they consider this a significant factor behind the much smaller rise in land prices between 1850 and the middle of the twentieth century than in the period since then. Rapid falls in passenger transport costs (reflecting rises in average speed) substantially increased the available quantity of land within a given travel time of large urban centers in the period 1850-1940; such increases have been far smaller in the period since.

To understand how housing markets develop over time we use a model with the following features. Houses are constructed by profit maximizing firms combining land at different locations with structures using a technology which allows substitutability between the two types of input. Households make decisions on location, and consumption of goods and housing, which generate regional differences in the types of house built and the relative prices of houses to consumer goods. We assume that there are advantages of being close to central urban areas - wages may be higher there; amenities better; availability of goods greater. We do not analyze why this might be true and take it as a given. (We note however the extensive evidence consistent with this which has made it a standard assumption in a large literature following Krugman (1991). For a survey of the evidence see Combes & Gobillon (2014)). Density of population and of development across regions varies endogenously over time. The total supply of land is fixed but the extent to which land is used varies endogenously as the value of houses determines whether marginal land can be commercially developed. Aggregate production of goods can be used for immediate consumption or for investment in

productive capital and structures, which depreciate at different rates. Interest rates clear the market for saving and investment and vary over time.

The model is described in detail in Section 1. In section 2 we analyze equilibria. Section 3 describes how the model is solved and calibrated. Section 4 shows simulation results and their sensitivity to key parameters. Section 5 concludes.

1   The Model

1.1   The Dynasties

Our economy consists of a *continuum of dynasties* on the unit interval. Though the number of dynasties remains constant over time, the number of people in each dynasty grows at rate m which is therefore also the rate of population growth. If we normalize the size of each dynasty to be 1 at time $t = 0$, then the total population at time t, $n(t)$, is equal to e^{mt}. Each dynasty has command over the same initial endowment of resources, in the form of labor, L, capital, K, and land, R; the dynasties only differ in where they choose to live. Labor is supplied inelastically and in proportion to dynastic size at each period, and so the labor force grows at rate m too.

Each dynasty derives utility from the consumption of goods, denoted C, and of housing services, S. Preferences over these goods at a given time t is described by a constant elasticity of substitution (CES) utility function

$$Q_{it} = \left[aC_{it}^{1-1/\rho} + (1-a)S_{it}^{1-1/\rho} \right]^{1/(1-1/\rho)} \tag{1}$$

where ρ is the elasticity of substitution between housing and consumption goods and a is a share parameter. The indices i and t index the quantity to dynasty i at time t. Dynastic welfare is the discounted power function of instantaneous utility

$$W_{i0} = \int_0^\infty \frac{1}{1-\gamma} Q_{it}^{(1-\gamma)} e^{-\theta t} dt \tag{2}$$

where γ reflects the degree of inter-temporal substitutability and θ is the discount factor. The consumption good is the numeraire, the real interest rate is r_t and the price of housing services at location l at time t is p_{lt}. We shall shortly describe the model's geography.

Dynasties maximize their welfare (2) given their endowment and prices. We use a dynasty as our decision making unit throughout. We could equally have done the analysis in per capita terms. If we assume that the flow of dynastic utility at time t is the sum of utilities of identical dynastic members then alive - whose number is proportional to $n(t)$ - then because the utility function is constant returns to scale (CRS) and population growth

is constant the welfare function in (2) can be re-written as

$$W_{i0} = \int_0^\infty \frac{1}{1-\gamma} \left[a \left(\frac{C_{it}}{n(t)} \right)^{1-1/\rho} + (1-a) \left(\frac{S_{it}}{n(t)} \right)^{1-1/\rho} \right]^{\frac{(1-\gamma)}{(1-1/\rho)}} n(t) e^{-\tilde{\theta}t} dt \qquad (3)$$

where now the discount rate $\tilde{\theta} = \theta + \gamma m$. Thus the dynastic welfare function (2) is equivalent to a welfare function that is the sum over members of the dynasty of their individual utilities but with a shifted discount factor.

We use the same notation to denote aggregate quantities but without the index i. These aggregate quantities are just the sum over dynasties of the respective quantities; thus the aggregate consumption of housing services at time t, S_t, is

$$S_t = \int_0^1 S_{it} di \qquad (4)$$

1.2 Goods Production Sector

The production side of the economy consists of 2 sectors; a goods production sector and a housing production sector. The goods production sector uses Cobb-Douglas technology, F, to manufacture the single good. This good can be consumed, C, or invested in productive capital, I_K, or in residential buildings, I_B. We assume a constant rate of labor augmenting technical progress, g. Thus production in the goods sector is

$$C_t + I_t^K + I_t^B = F \left(K_t, L_t e^{gt} \right) = A K_t^\alpha \left(L_t e^{gt} \right)^{1-\alpha} \qquad (5)$$

where α is the capital share of output. The stock of capital, K, and residential buildings, B, evolve over time as

$$\dot{K}_t = I_t^K - \delta^K K_t \qquad (6)$$

$$\dot{B}_t = I_t^B - \delta^B B_t \qquad (7)$$

where δ^K and δ^B are the respective constant depreciation rates of productive capital and residential buildings.

For the production sector to be in equilibrium, the net return to capital must equal the real interest rate, that is

$$r_t = \frac{\partial F}{\partial K_t} - \delta^K = \alpha A \left(\frac{L_t e^{gt}}{K_t} \right)^{1-\alpha} - \delta^K \qquad (8)$$

1.3 Housing Sector

We assume a circular economy with a physical area given by a circle of radius $l_{\max}$ (where $\pi l_{\max}^2 = R$), with the central business district (CBD) located, unsurprisingly, at the center.

We adopt this monocentric assumption, though recognize that in actual economies there may be more than one central location. Despite the fact that the monocentric model has some obvious shortcomings - for example that land values do not decline monotonically in all directions form an urban center - it captures an important aspect of variation in land rents. Ahlfeldt (2011) finds that once allowance is made for various other geographic factors the monocentric model does a good job in explaining house and land price variability across space. The key assumption for us is not so much that there is only one such central location in each economy but rather that new centers will not emerge endogenously. It is striking that in many developed economies there have been very few big new cities that have emerged even over long periods like the last 200 years, (Glaeser & Kohlhase 2004).

We assume a simple cost of distance function levied on consumption. Thus, we follow a large literature (e.g. Krugman (1980)) in using Samuelson's 'Iceberg' model of transport costs; that a fraction of any good shipped simply 'melts away' in transit. Formally, at distance l from the CBD, $1 + \lambda_t l$ of consumption good must be purchased to consume 1 unit of the good. We think of $\lambda_t l$ as the tax on location with λ_t as the impact at time t of distance on that tax rate. There is no location tax on housing services in this formulation, though taxing them at the same rate as consumer goods makes no difference to the model (and is offset by slightly wider regional differences in house prices). In an economy where all agents are identical, a common distance tax on all consumption is isomorphic to an economy with a distance tax on labor, (see Atkinson & Stiglitz (2015)). Of these three slightly different formulations - a distance tax on consumption of goods only, a tax on all consumption (including housing) or a labor tax - we stick with the traditional 'iceberg' formulation. However the results are almost identical across the three formulations.

There are many aspects of the cost of location and several interpretations of $\lambda_t l$. The most obvious is travel costs - you need to spend time and money on getting nearer to the center where you may work and where you can most easily buy goods and consume them. This idea goes back at least to von Thunen (Von Thünen & Hall 1966). Many goods need to be brought to location l at greater cost than being brought to places nearer the center and that to go to the center and buy them cost you time and travel expenses; this is in the spirit of Samuelson's iceberg costs of moving things and the net effect is that the costs of such goods is raised by $\lambda_t l$. (There is evidence that in the US goods are less expensive on average in cities than in other parts of the country - see Handbury & Weinstein (2014)). It is also consistent with Krugman's model of commuting costs, where all dynasties have a fixed supply of labor but lose a proportion of this supply in commuting to the CBD for work. One might also view $\lambda_t l$ as the cost of being less productive away from the center where economies of scale, positive externalities or network effects means that real wages and productivity are higher; there is extensive evidence for such agglomeration economies, (Combes & Gobillon 2014).

The cost of distance on all these interpretations is linked to transport costs. There is no reason to think that $\lambda_t l$ is constant over time - it reflects technology (most obliviously

travel technology) which has changed a lot. We come back to that later.

Dynasties have the same preferences and endowments but differ in where they choose to live. As dynasties are otherwise identical and markets competitive, prices adjust so that dynasties are indifferent to where they locate - the cost of distance for each dynasty being offset exactly by lower housing costs further from the center.

The housing of the dynasty at location l at time t is provided by combining buildings[2], B_{lt}, and land, R_{lt}, at location l. The same CES technology is used at all locations. The flow of services from a given amount of housing is assumed to be proportional to the amount of housing – both of which are the result of combining structure and land. Therefore, up to a constant the stock of housing and flow of housing services are the same. We write the relation:

$$S_{lt} = H(B_{lt}, R_{lt}) = A_s \left[b B_{lt}^{1-1/\varepsilon} + (1-b) R_{lt}^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} \tag{9}$$

where ε is the elasticity of substitution between land and structure and b the share parameter in the housing market. A_s is the factor of proportionality between units of housing and the flow of services. B_{lt}, R_{lt} are the use of structures and land for the dynasty located at distance l at time t. As land is cheaper further away from the CBD, the mix of land to buildings will increases the further away from the center the house is located. To calculate how this mix changes with distance, we first calculate the price of housing services so that dynasties are indifferent to where they locate.

Let p_{lt} be the price of housing services at location l at time t. This is the user cost of housing - the rent that would have to be paid at time t for a unit of housing services at location l; it is also the opportunity cost of having wealth in the form of such housing. If the dynasty is to be indifferent between living at the center relative to any other location, the price of housing services must move so that for the same level of expenditure (including transport costs), the dynasty achieves the same level of utility wherever they are located. Hence, if we denote the level of expenditure of each dynasty at time t as E_t and the indirect utility at location l as Q_{lt}^* then

$$Q_{lt}^* = \sup_{C_{lt}, S_{lt}} \left(a C_{lt}^{1-1/\rho} + (1-a) S_{lt}^{1-1/\rho} \right)^{1/(1-1/\rho)} \tag{10}$$

given that

$$(1 + \lambda_t l) C_{lt} + p_{lt} S_{lt} \leq E_t.$$

For equilibrium we require $Q_{lt}^* = Q_{0t}^*$ for all l. The indirect utility function, (10), is therefore

$$Q_{lt}^* = \frac{a^{1/(1-1/\rho)} E_t}{(1 + \lambda_t l)} \left(1 + \left(\frac{p_{lt}}{(1 + \lambda_t l)} \right)^{1-\rho} \left(\frac{(1-a)}{a} \right)^{\rho} \right)^{1/\rho/(1-1/\rho)} \tag{11}$$

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[2]This is a slight abuse of notation. By B_{lt}, R_{lt} we mean the amount of land and structure used in the house of the dynasty whose location at time t is centered at distance l.

which is achieved at location l when

$$C_{lt} = \frac{E_t}{(1 + \lambda_t l) \left[1 + \left(\frac{p_{lt}}{(1+\lambda_t l)} \right)^{1-\rho} \left(\frac{(1-a)}{a} \right)^{\rho} \right]}, \qquad S_{lt} = \frac{E_t}{(1 + \lambda_t l) \left[\left(\frac{a}{(1-a)} \right)^{\rho} + \left(\frac{p_{lt}}{(1+\lambda_t l)} \right) \right]}.$$

$$(12)$$

If the market is to be in equilibrium, then costs of housing must be such that at the given expenditure E_t the indirect utility is equal at each location. For that to hold we require that the price, p_{lt}, be the following function of the price at the center, p_{0t},

$$p_{lt} = \left(p_{0t}^{1-\rho} - \left(\frac{a}{1-a} \right)^{\rho} \left((1 + \lambda_t l)^{1-\rho} - 1 \right) \right)^{1/(1-\rho)} \qquad (13)$$

In the spatial geography literature, this is often referred to as the bid-price function. Here we have been able to derive a bid-price function for CES preferences given our particular model of transport costs.

Prices must also be non-negative. When $\rho < 1$, housing will only be built at location l if

$$l \leq \frac{1}{\lambda_t} \left(\left(p_{0t}^{1-\rho} \left(\frac{(1-a)}{a} \right)^{\rho} + 1 \right)^{1/(1-\rho)} - 1 \right) \qquad (14)$$

We can now substitute for p_{lt} in the indirect utility function, equation (11), using equation (13). Thus the indirect utility function, Q_{lt}^*, at each location is

$$Q_{lt}^* = \frac{E_t}{\left((1-a)^{\rho} p_{0t}^{1-\rho} + a^{\rho} \right)^{1/(1-\rho)}}. \qquad (15)$$

This is the other advantage of using the 'Iceberg' model of transport costs; that the indirect utility does not have to expressed as a function of location, l, and prices at a given location, p_{lt}, as per the Alonso-Mills-Muth urban model but can be written as a function of the cost of housing at the center, p_{0t} and expenditure, E_t. To recognise this, we therefore drop the subscript l and denote this indirect utility function as $Q_t^*(E_t, p_{0t})$. Thus all dynasties not only have the same utility whatever their location (this is the equilibrium condition that determines the price of housing services), but also have the same marginal utility of expenditure. As all dynasties also have the same labor endowment and face the same interest rate and wage, they will all chose the same total expenditure path over time if they start with the same initial wealth - though the split between spending on housing and on consumption at each point in time will differ by location. We can therefore solve for the optimal path of spending for a dynasty living at the center, and know that every other dynasty will choose the same expenditure path though they choose to live at different locations and consume a different mix of housing and consumption goods. It is this condition that makes it straightforward to solve for the full dynamic general equilibrium.

It is central to this model that the price of consumption varies with location; at location

l the price is $(1 + \lambda_t l)$. So when deciding on their intertemporal allocation of consumption, a dynasty must consider the post transport cost of consumption which is weighed against the marginal increase in utility from reallocating resources to time t. An application of the Envelope theorem[3] to the optimsation problem in (10) shows that this marginal utility

$$\frac{\partial Q_t^*}{\partial E_t} = \frac{\partial Q_{lt}}{\partial \left((1 + \lambda_t l) C_{lt} \right)} = \frac{1}{(1 + \lambda_t l)} \frac{\partial Q_{lt}}{\partial C_{lt}}, \tag{16}$$

is the same at every location l and equal to the marginal utility of expenditure.

An equilibrium condition in the housing sector is that all investment in residential buildings must earn the real rate of return, r_t. This condition sets the mix of residential buildings to land at each location l[4]. Given the rental rates p_{lt} at each location, the real return to buildings at location l is

$$r_t = \left(p_{lt} \frac{\partial H \left(B_{lt}, R_{lt} \right)}{\partial B_{lt}} - \delta_B \right) \tag{17}$$

Given the housing technology (9), then this condition combined with the demand for housing services in (12) gives the demand for residential buildings at location l

$$B_{lt} = \left(\frac{p_{lt} A_s^{1-1/\varepsilon} b}{r_t + \delta_B} \right)^\varepsilon S_{lt} \tag{18}$$

The condition (17) also implies the mix of buildings to land must satisfy

$$\left(\frac{r + \delta_B}{p_{lt} b A_s} \right)^\varepsilon = \left(b + (1 - b) \left(\frac{R_{lt}}{B_{lt}} \right)^{1-1/\varepsilon} \right)^{1/(1-1/\varepsilon)}. \tag{19}$$

Hence given the demand for buildings in (18), this equation determines the size of the residential plot, R_{lt}. Thus from the interest rate and optimality condition (12), we can back out the implied demand for both buildings and land at each location.

As rental rates, p_{lt}, fall the further we are from the center, equation (19) implies that the ratio of land to buildings rises. Substituting out for the rental rate using equation (13), one can alternatively express this ratio as a function of the rental rates at the center and location l. If $\varepsilon < 1$ (and there is a great deal of empirical evidence to suggest it is, see below) then for a large enough country there comes a distance from the center when it is no longer possible to earn a return r_t on residential building - even when combined with an infinite amount of land. If this distance is less than the radius of the economy, then the

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[3] Alternatively, and effectively equivalently, one can take the partial derivative of (1) with respect to consumption and substituting in for the optimal choice of consumption at location l in equation (12) and rearrange with reference to equations (13) and (15).

[4] In some of the simulations discussed later, we place restrictions on the maximum buildings to land ratios (so as to model planning restrictions). In these cases the following optimal allocation condition is not satisifed. Instead, as both the prices of housing services and the building to land ratios are given, buildings recieve rB_{lt} of the rental income with land receiving the residual.

condition that $(r + \delta_B) > 0$ ties down the edge of residential development at time t. Thus the edge of the inhabited region of the economy at time t, denoted $l_{t,\text{Edge}}$, is given by

$$l_{t,\text{Edge}} = \min\left(l_{\max}, \frac{1}{\lambda_t}\left(\left(1 + \left(p_{0t}^{1-\rho} - \left(\frac{(r_t + \delta_B)}{A_s b^{1/(1-1/\varepsilon)}}\right)^{1-\rho}\right)\left(\frac{(1-a)}{a}\right)^{\rho}\right)^{1/(1-\rho)} - 1\right)\right)$$

(20)

For $\varepsilon < 1$, this condition is tighter than the condition for the positive rental price in equation (14). For $\varepsilon > 1$ the opposite is true. In this case the limits to the urban sprawl are determined by (14), that is

$$l_{t,\text{Edge}} = \min\left(l_{\max}, \frac{1}{\lambda_t}\left(\left(p_{0t}^{1-\rho}\left(\frac{(1-a)}{a}\right)^{\rho} + 1\right)^{1/(1-\rho)} - 1\right)\right).$$

(21)

To complete the description of the housing sector, we consider the price at time t of land at a distance l from the center (p_{lt}^{Land}). At all points land needs to generate a return equal to the real interest rate. This implies that the rent on the land, $p_{l\tau}\frac{\partial H(B_{l\tau}, R_{l\tau})}{\partial R_{l\tau}}$ plus capital gains equals the interest rate,

$$p_{l\tau}\frac{\partial H(B_{l\tau}, R_{l\tau})}{\partial R_{l\tau}} + \dot{p}_{lt}^{Land} = r_t p_{lt}^{Land}.$$

(22)

Integrating this relationship forward implies that the price of land is the discounted value of all future land rents, that is

$$p_{lt}^{Land} = \int_t^\infty e^{-\int_t^\tau r_v dv} p_{l\tau}\frac{\partial H(B_{l\tau}, R_{l\tau})}{\partial R_{l\tau}} d\tau$$

(23)

$$= \int_t^\infty e^{-\int_t^\tau r_v dv} p_{l\tau}(1-b)A_s \left(b\left(\frac{B_{lt}}{R_{lt}}\right)^{1-\frac{1}{\varepsilon}} + (1-b)\right)^{\frac{1/\varepsilon}{(1-1/\varepsilon)}} d\tau$$

(24)

Equations (17) and (22), along with the CES production function for housing services, also allow us to write the user cost of housing as

$$\begin{aligned}p_{lt} &= p_{lt}\frac{\partial H(B_{lt}, R_{lt})}{\partial B_{lt}}\left(\frac{B_{lt}}{S_{lt}}\right) + p_{lt}\frac{\partial H(B_{lt}, R_{lt})}{\partial R_{lt}}\left(\frac{R_{lt}}{S_{lt}}\right) \\ &= (r_t + \delta_B)\left(\frac{B_{lt}}{S_{lt}}\right) + \left(r_t p_{lt}^{Land} - \dot{p}_{lt}^{Land}\right)\left(\frac{R_{lt}}{S_{lt}}\right)\end{aligned}$$

(25)

Thus the user cost of housing is equal to the interest rate on assets per unit of housing plus depreciation on buildings minus any capital gains on the land.

To evaluate aggregate quantities in the housing sector, we need to be able to integrate across all locations rather than over dynasties. We therefore define a mapping, $i_t(l)$, that identifies the dynasty living at location l at time t. We let $i_t(0) = 0$ at the center. At radius l, the area of residential land in an annulus of width dl is $(2\pi l)\,dl$. The number of dynasties living in this annulus is equal to this area divided by the size of the residential land plot at

this location, R_{lt}. As dynasties are identical, except for location, the ordering of dynasties is unimportant. We shall therefore assume the ordering implicit in the following differential relationship is satisfied

$$di_t(l) = \left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{26}$$

Using this relationship, we can calculate the total demand for consumption goods, housing services and residential buildings (residential capital) by integrating over the inhabited area of the economy. Thus aggregate consumption of housing services given in equation (4) is calculated as

$$S_t = \int_0^1 S_{it}di = \int_0^{l_{t,\text{Edge}}} S_{lt}\left(\frac{2\pi l}{R_{lt}}\right) dl \tag{27}$$

and similarly for the aggregate residential building stock

$$B_t = \int_0^1 B_{it}di = \int_0^{l_{t,\text{Edge}}} B_{lt}\left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{28}$$

The aggregate value of land wealth (which we denote by LW) is given by

$$LW_t = \int_0^{l_{\max}} p_{lt}^{Land}\left(2\pi l\right) dl. \tag{29}$$

We can now define the average price of housing, p_t^{House}. This is the total value of housing - the sum of land and structures - divided by the stock of housing. Hence the price of housing is defined as

$$p_t^{House} = \frac{(LW_t + B_t)}{S_t} \tag{30}$$

Note from equation (9), that S_t is proportional to the stock housing and this makes p_t^{House} a valid index of the price of housing.

Aggregate consumption of goods must include the sum of all dynastic consumption which includes the distance tax or consumption 'melt'. Hence aggregate consumption is

$$C_t = \int_0^{l_{t,\text{Edge}}} (1 + \lambda_t l)C_{lt}\left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{31}$$

And finally the number of housed dynasties is

$$D_t = \int_0^{l_{t,\text{Edge}}} \left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{32}$$

(It will be an equilibrium condition that $D_t = 1$ for all t).

These aggregate variables need to be consistent with the production sector for the economy to be in equilibrium.

2 Equilibrium Conditions

Given an initial total capital stock, $K_0 + B_0$, an equilibrium exists if for all t there exists positive finite prices r_t, w_t and p_{0t} (the real interest rate, wage rate and rental price of housing at the center) that supports a competitive equilibrium. We first state conditions for equilibrium on the demand side.

2.1 Demand Side Equilibrium

Given these prices we first describe the path of consumption for dynasty 0, which is assumed to live at the center $l = 0$. This dynasty, like all others, has an initial wealth (excluding human capital) equal to aggregate physical capital divided by the number of dynasties. Aggregate capital at that initial period is $K_0 + B_0$ while the value of land is LW_0 ; the dynasties have unit mass so each dynasty's initial wealth is equal to $K_0 + B_0 + LW_0$ The following conditions must hold:

1. Dynasty 0 has consumption and housing demands that satisfy the intertemporal budget constraint

$$K_0 + B_0 + LW_0 + \int_0^\infty e^{-\int_0^t r_\tau d\tau} w_t e^{mt} dt = \int_0^\infty e^{-\int_0^t r_\tau d\tau} \left(C_{0t} + p_{0t} S_{0t} \right) dt, \quad (33)$$

2. the intertemporal efficiency condition holds

$$\frac{\frac{d}{dt}\left(Q_{0t}^{-\gamma} \frac{\partial Q_{0t}}{\partial C_{0t}} \right)}{Q_{0t}^{-\gamma} \frac{\partial Q_{0t}}{\partial C_{0t}}} = \theta - r_t \quad (34)$$

3. and the intratemporal efficiency condition holds

$$p_{0t} = \frac{\partial Q_{0t}}{\partial S_{0t}} \bigg/ \frac{\partial Q_{0t}}{\partial C_{0t}} = \frac{(1-a)}{a} \left(\frac{C_{0t}}{S_{0t}} \right)^{1/\rho} \quad (35)$$

for all t.

Given the convexity of the utility function, prices r_t, w_t and p_{0t} generate a unique path for consumption of goods and housing for the dynasty located at the center (C_{0t} and S_{0t}) that satisfy these efficiency conditions and budget constraint.

Denote the total expenditure of dynasty 0 along this path as $E_t = (C_{0t} + p_{0t} S_{0t})$. At each point in time the consumption bundle (C_{0t}, S_{0t}) solves the instantaneous optimisation problem (10) at time t and location 0. Thus the acheived utility is given by the indirect utility function $Q_t^*(E_t, p_{0t})$ in (15). We can therefore rewrite the intertemporal optimisation (33), (34) and (35) in terms of the prices r_t, w_t, p_{0t}, expenditure E_t and utility function Q_t^*. This restatement is independent of the location of the dynasty. Thus all dynasties follow an identical total expenditure path; though the composition of their consumption

bundle, (C_{lt}, S_{lt}), will depend on their location. This bundle will solve their intratemporal efficiency condition at location l

$$\frac{p_{lt}}{(1 + \lambda_t l)} = \frac{\partial Q_{lt}}{\partial S_{lt}} \Big/ \frac{\partial Q_{lt}}{\partial C_{lt}} = \frac{(1 - a)}{a} \left(\frac{C_{lt}}{S_{lt}}\right)^{1/\rho} \tag{36}$$

where rental prices at location l are given by equation (13)

$$p_{lt} = \left(p_{0t}^{1-\rho} - \left(\frac{a}{1-a}\right)^{\rho} \left((1 + \lambda_t l)^{1-\rho} - 1\right) \right)^{1/(1-\rho)}.$$

The solution to this problem is given earlier in equation (12). Aggregating over all housed dynasties gives paths for aggregate consumption, equation (31), aggregate demand for residential buildings, equation (28), and number of housed dynasties, equation (32). These aggregate quantities need to satisfy the conditions for a supply side equilibrium.

2.2 Supply Side Equilibrium

Given the initial total capital stock, $K_0 + B_0$, and the aggregate path for residential buildings, B_t, consumption goods, C_t and labor, L_t, then the first order differential equation implied by production goods technology equation (5),

$$\dot{K}_t + \dot{B}_t = AK_t^{\alpha} \left(L_t e^{gt}\right)^{1-\alpha} - \delta_K K_t - C_t - \delta_K B_t \tag{37}$$

implies a unique path for the capital stock K_t. This path of these aggregate variables must be supported by the equilibrium prices r_t, w_t and p_{0t}. The supply side conditions for a competitive equilibrium are that

1. Wages satisfy the profit maximizing condition of firms and can be expressed in terms of the interest rate

$$w_t = (1 - \alpha) A^{1/(1-\alpha)} \left(\frac{\alpha}{r_t + \delta_K}\right)^{\alpha/(1-\alpha)} \tag{38}$$

2. net returns to capital and to residential buildings are equal to the interest rate

$$r_t = \frac{\partial F}{\partial K_t} - \delta^K = \alpha A \left(\frac{L_t e^{gt}}{K_t}\right)^{1-\alpha} - \delta^K \tag{39}$$

$$r_t = \left(p_{lt} \frac{\partial H(B_{lt}, R_{lt})}{\partial B_{lt}} - \delta_B\right) \tag{40}$$

3. the production of housing services (the number of housed dynasties) is equal to the demand for housing services (the number of dynasties to be housed)

$$1 = D_t = \int_0^{l_{t,Edge}} \left(\frac{2\pi l}{R_{lt}}\right) dl. \tag{41}$$

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are satisfied for all t. As condition (40) is satisfied by the construction of an equilibrium in the housing sector, the three conditions (38), (39) and (41) imply a unique solution for the three prices r_t, w_t and p_{0t}.

2.3 Balanced Growth Path

Effective labor supply grows at the sum of the rate of productivity plus population growth, $g + m$. We now show if the location tax, λ_t (which is predominantly a proxy for the costs of travel) falls at half this rate, $(g + m)/2$, then, as long as the urban expansion does not approach the edge of the country, $l_{t,\text{Edge}} < l_{\max}$, the economy will tend toward a balanced growth path where all economic aggregate quantities grow at the rate $g + m$ and the average prices of land and houses are constant. Our model, therefore, admits a balanced growth path, even though one of the factors is land and is in fixed supply. This is because it is not efficient to use all available land - the costs of travel make in uneconomic to use the land further than $l_{t,\text{Edge}}$ from the centre - but as the costs of travel fall it becomes economically viable to use more land, effectively increasing its supply. If improvements in travel technology imply the cost of travel falls at exactly the rate $(g + m)/2$, then the usable area of land increases at rate $(g + m)$ as does the size of the economy.

We shall now demonstrate that the economy converges to a steady-state balanced growth path if the location tax declines at a rate of $(g + m)/2$. We do so by showing that in a country of infinite size, $(l_{\max} = \infty)$, there exists a balanced steady state if the location tax falls at $(g + m)/2$. We then appeal to a Turnpike Theorem to argue that when a country is of finite size, the economy will hug this optimal path until the urban expansion approaches the edge of the country.

We proceed by assuming a steady state and then verify that this satisfies the equilibrium conditions. Assume for all time $t \geq t_0$, supporting prices are constant; $r_t = r_{t_0}$, $w_t = w_{t_0}$ and $p_{0t} = p_{0t_0}$ and aggregate quantities all grow at rate $(g + m)$. If this is to be consistent with the equilibrium condition (34), then the interest rate r_t satisfies the balanced growth path (or Ramsey) condition

$$r_t = r_{t_0} = \theta + \gamma (g + m). \tag{42}$$

To describe the spatial economy along the balanced growth path, introduce the scaled location variable, $\widetilde{l}(l, t) = l e^{-(g+m)(t-t_0)/2}$ which we also denote as $\widetilde{l}_t$ for short. Along the balanced growth path, this scaling maps the growing urban area back onto the urban area at t_0. This enables us to describe all variables in the spatial economy for $t \geq t_0$ in terms of their values at t_0. For example, as prices of housing at the centre are constant, $p_{0t} = p_{0t_0}$, and $\lambda_t = \lambda_{t_0} e^{-(g+m)(t-t_0)/2}$, then equation (13) implies that $p_{lt} = p_{\widetilde{l}_t t_0}$; that is the price of housing at time t and location l is equal to the price of housing at time t_0 and location $\widetilde{l}_t$. Similarly for the edge of the urban area, equations (20) or (21) imply that $l_{t_0,\text{Edge}} = \widetilde{l}(l_{t,\text{Edge}}, t)$; thus the urban area at time t maps back onto the urban area at time t_0. As distances are scaled back by $e^{-(g+m)(t-t_0)/2}$, areas will be scaled back by the square

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of this; hence we can write $R_{\widetilde{l}_t t_0} = R_{lt} e^{-(g+m)(t-t_0)}$. Under these definitions we show that the equilibrium conditions are satisfied at t if they are satisfied at t_0.

Eequilibrium condition (41) says that demand for land equals the supply of land. Assume this is satisfied at t_0. Then at time t we can map all quantities onto their respective values at t_0 to show that

$$\int_0^{l_{t,\mathrm{Edge}}} \left(\frac{2\pi l}{R_{lt}}\right) dl = \int_0^{l_{t_0,\mathrm{Edge}}} \left(\frac{2\pi \widetilde{l}_t}{R_{\widetilde{l}_t t_0}}\right) d\widetilde{l}_t = 1$$

and so (41) is satisfied at time t. Similarly let $C_{lt} = C_{\widetilde{l}_t t_0} e^{(g+m)(t-t_0)}$, $S_{lt} = S_{\widetilde{l}_t t_0} e^{(g+m)(t-t_0)}$ and $B_{lt} = B_{\widetilde{l}_t t_0} e^{(g+m)(t-t_0)}$ and substitute into equations (31), (28) and (27) respectively to show that this implies $C_t = C_{t_0} e^{(g+m)(t-t_0)}$, $S_t = S_{t_0} e^{(g+m)(t-t_0)}$ and $B_t = B_{t_0} e^{(g+m)(t-t_0)}$.

Given the spatial economy is consistent with a balanced growth path, it is trivial to show the rest of the economy is too. As both aggregate consumption and housing grow at the rate $(g+m)$ then equations (34) and (40) are satisfied at t as they are satisfied at t_0. Finally as aggregate capital K is also growing at rate $g+m$, then the production constraint (37) as well as the budget constraint (33) are all satisfied at t as they are satisfied at t_0.

Thus in a country of infinite size, there exists a steady state balanced growth path. For a country of finite size, it will be optimal to converge towards this path as close as possible for as long as possible before the constraint of the fixed factor (land) forces the economy to significantly diverge from this path. Our numerical simulations have exactly this property for the special case when λ_t falls at the rate $(g+m)/2$.

2.4   Solution Technique

We solve a discrete time approximation to the continuous time model described in this paper using the relaxation approach first described in Laffargue (1990) and Boucekkine (1995). We let a period be a year, so that $t = 0, 1, 2... T$ and solve for B_t and K_t at these points. To demonstrate that the path for these state variables describes the complete growth path for of economy, note that given such a path one can calculate investment I_t^K and I_t^B from discrete time equivalents of equations (6) and (7) respectively and C_t from the production equation (5) and then finally solve for the housing sector as described in 1.3.

To be able to solve for the economy in this way, we need a terminal condition for these state variables at $t = T+1$ in order to calculate I_t^K and I_t^B at $t = T$. To choose these terminal values, we appeal to the Turnpike theorem of McKenzie (1976). We can assume any reasonable values for the state variables at $T+1$; for the optimal growth path between our initial condition and this terminal value will 'hug' the optimal growth path of the infinite horizon problem as closely as possible for as long as possible before deviating off to the given terminal value. We therefore assume that $K_{T+1} = B_{T+1} = 100$ for a very large T and only report the growth path for $t \ll T$[5].

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[5]In practise we solve the model for 450 annual periods, and report answers only for the first 250. We checked that the path over these first 250 periods was different by less than 10^{-5} if we solved the model instead over 600 periods.

We are solving for $2(T+1)$ unknowns; B_t and K_t for $t = 0, 1, 2... T$ and therefore need to $2(T+1)$ constraints. The first constraint is the initial condition, that $B_0 + K_0$ equals initial stock of capital. A further T constraints stem from the dynamic efficiency condition, equation (34), at $t = 1, 2... T$. The other $(T+1)$ constraints are that the marginal product of residential buildings equals the marginal product of reproductive capital, equation (17) and (8) at $t = 0, 1, 2... T$. Hence, along the optimal path, the model variables as described by that path of B_t and K_t must solve these $2(T+1)$ constraints. We write these conditions as a set of non-linear equations denoted $f(B_t, K_t) = 0$ where f is a $2(T+1)$-vector.

The relaxation approach starts from an initial guess for the path of the state variables[6]. It then uses a standard Newton-Raphson iterative procedure to solve the $2(T+1)$ non-linear equations. We set the convergence condition to a change of less than 10^{-6} between iterations. To achieve this level of accuracy took no longer than a couple of minutes of a Intel Core i5 2.7GHz processor .

3 Calibration

We outline how we set key parameters. Since the distance cost parameter is central to the model, and the facts on its evolution are less familiar and clear cut than for other parameters, we first consider the empirical evidence on transport costs in some detail.

3.1 Transport costs

λ reflects the cost of living further from the (economic) center of the country - it is essentially a tax on distance. The most natural measure of that tax is the cost of travelling to the center. Some of that cost is the price of a rail, bus or plane ticket or of fuel to drive a car. Much of it is in the form of time taken to get to work or to get to shops, restaurants, or theaters. It is clear that this cost, and its rate of change, vary substantially over time. The development of railways in the second part of the nineteenth century dramatically brought down the time cost of commuting. Increased use of cars in the twentieth century brought cost down further. There have been great increases over the twentieth century in the average distance people travel to work, a phenomenon linked to urban sprawl and made possible by improvements in transport infrastructure and technology. But the rate of improvement in travel speeds has slowed in the past fifty years - on some measures it has stopped completely. It is also likely that the cost of moving goods has fallen by far more than the cost of moving people.

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[6]This guess is not critical. However our initial guess was constructed by assuming $K_t/B_t = 3$ for all t. We then set K_t equal to

$$K_t = \left(K_0 - \left(K_0 - \frac{K_{T+1}}{e^{g(T+1)}} \right) \frac{t}{T+1} \right) e^{gt}$$

where K_0 was chosen so that the initial condition was satisified and K_{T+1} was equal to our chosen terminal value for K_t. This guess was good enough to ensure quick convergence.

There is an extensive literature on the evolution of passenger travel over time focusing on average distances travelled in a day, average time spent travelling and the implied average speed at which people travel. There is a good deal of evidence that average time spent traveling is surprisingly constant over time and across countries at about 1 hour a day. Ausubel et al. (1998), Schafer (2000), Schafer & Victor (2000), Levinson & Kumar (1994) all present evidence that, a least up to the 1990s, there has been near constancy of travel times, supporting a hypothesis originally put forward by Zahavi (Zahavi & Talvitie (1980)). Figure 1 of Schafer & Victor (2000) is particularly striking revealing a clustering of average daily travel times at around 1 hour for countries of very different standards of living and across different periods up to the early 1990s.

It is the inverse of average travel speeds that we take as our measure of the cost of distance. Ausubel et al. (1998) show estimates of the average daily distance travelled by US citizens over the period 1880 to 1998. If time spent traveling is roughly constant this is a measure of the evolution of average travel speed. The average annual rise over the whole period is 2.7%, but average travel times have increased a little over time so 2.7% is likely an over-estimate of the rise in passenger travel speeds. The data also show a clear falling off in the growth of travel distances from about 1970 (Ausubel et al. (1998, Figure 3)). The growth in distance is at its fastest between around 1900 and 1930.

Glaeser & Kohlhase (2004) present evidence of increasing travel times to work for all major US cities between 1980 and 2000. They conclude "...that people-moving costs are not declining within US cities." This is in marked contrast to what had happened in the first half of the twentieth century and, especially, in the second half of the nineteenth century.

Detailed data on travel speeds is rather patchy, especially for the late nineteenth and early twentieth century. We rely on surveys from various industrial economies to piece together a plausible path for the overall evolution of travel speeds and use this to tie down the rate of change of λ.

Survey evidence suggests that in the UK in the late nineteenth century around 60% of journeys people undertook were made on foot (Pooley & Turnbull (1999), Table 5). Fifty years earlier - and before the spread of railways - the proportion walking would have been much higher. By 1920 a substantial proportion (around 50%) used trains or buses for journeys and the proportion walking had fallen to under 30%. For those travelling to London trains and buses accounted for close to 80% of journeys over 1920-29 (Pooley & Turnbull (1999), table 6). They report average journey speeds for Britain in the period 1920-1929 of 13.9 kilometers per hour (Table 3). In 1850 it is likely that this average speed was not much more than 4-5 kilometers per hour. Something that is plausibly close to a tripling of average travel speeds took place in the seventy years between 1850 and the mid 1920's. Leunig (2006) reports an enormous (16 fold) increase in distances travelled in the UK between 1865 and 1912 and very large improvements in average travel speeds. In the next 70 years, according to the data reported in Pooley & Turnbull (1999), travel speeds rose by about 80%; over this period car use rises greatly but the improvement in average

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travel speeds is very substantially smaller than when railways and buses displaced walking in the earlier period. For those travelling into London, which is clearly the economic center of Britain, improvements in average travel speeds between 1925 and 2000 was markedly slower than the average for all journeys in the UK.

Based on this UK historical data the annual improvement in speeds over the period 1850-1925 was a little under 1.5% a year while over the period 1925-2000 it dropped to about 0.8% a year.

This substantial slowdown in the improvement in travel speeds is borne out by data from many developed countries. Ausubel et al. (1998) shows something similar for the US where walking and travel by horse account for a sharply falling share of travel over time, replaced first by trains and later by cars. The common pattern of the spread of railways is one factor behind a general and sharp rise in speeds in the later part of the nineteenth century. Railroad density (kilometers of railways per 1000 people) show enormous expansion in Germany, Britain and the US between 1850 and 1920, when railroad density peaked, (Hugill 1995). More recently, and now that car journeys have become the most used means of travel in developed countries, improvements in travel speeds have fallen. Evidence of flat or even rising travel times into large cities exists for many countries: Glaeser & Kohlhase (2004) present evidence of rising congestion and of travel times into major US cities since 1990; van Wee et al. (2006) present evidence of increasing travel times in Holland. In considering the future evolution of house prices it may be plausible to consider that λ is now constant.

For the base case we assume λ falls by 1.5% a year between 1870 and 1945, then falls by 0.8% a year until 1980 and is then constant.

3.2 Other parameters

For some parameters we can take guidance from values that are implied by steady state growth paths. As we argue below, the evolution of productivity, population and transport costs mean that for many developed economies the condition for steady state growth (λ falls at the rate $(g + m)/2$) may have approximately held for roughly the 100 years 1860-1960 and so key aggregate ratios for that period help in calibration.

m : US population rose at an average annual rate of around 1.2% in the 100 years from 1911 to 2011. It has grown slightly more slowly in the 50 years to 2011 at rate of around 1.1% a year. We set population growth at 1.0% a year.

g : We set g at 0.02 based on historical long run growth in productivity in many developed economies of around 2%.

δ_K : On a steady state growth path $\delta_K = (I^k/K) - (m+g)$. Using US data on the average ratio of non residential capital investment to the non residential capital stock since 1929, and using the values of $m + g$ as above, implies a depreciation rate of just above 6%. For the US Davis & Heathcote (2005) use a quarterly value for depreciation of business capital of 0.0136 (annual of around 5.4%). Kiyotaki et al. (2011) use 10%. We set depreciation at

7% a year.

$\boldsymbol{\delta_B}$: As with nonresidential capital, on a steady state growth path the depreciation of residential capital is given by $\delta_B = (I^B/B) - (m+g)$ Using US data on the average ratio of residential capital investment to the residential capital stock since 1929, and using the values of $m+g$ as above, implies a depreciation rate of only around 1.25% a year. This seems slightly lower than estimates based on the difference between gross and net US residential investment which gives a a figure near 2%. Fraumeni (1997) reports that in the US structures depreciate at a rate between 1.5% and 3% a year. Van Nieuwerburgh & Weill (2010) use 1.6% in their simulations. Davis & Heathcote (2005) use a quarterly value for depreciation of the housing stock of 0.0035 (annual of around 0.014). Hornstein (2009) suggests a figure of 1.5%. We set depreciation on residential structures at 2% a year.

$\boldsymbol{\gamma}$: There is much evidence that the degree of inter-temporal substitutability is less than 1. Hall (1988) estimated it was close to zero. Subsequent work suggests a significantly higher value, but still less than unity (see Ogaki & Reinhart (1998) and Vissing-Jørgensen (2002)). We set the intertemporal elasticity to 0.67 which implies $\gamma = 1.5$.

$\boldsymbol{\theta}$: On a steady state path $r = \theta + \gamma(m+g)$. Given the values used for γ, m, g we can use this relation to gauge a plausible value of θ conditional on an assumed value for the steady state rate of return. In our model all assets (land, nonresidential capital, structures, housing) generate the same return. In practice assets obviously do not generate the same average returns. Jordà et al. (2017) provide data on the real returns on a range of assets (including equities, bonds and housing) over the period 1870-2015 for 16 advanced economies. Returns on equities and housing look similar and average about 7% a year - though they are a little lower pre-1950. Bonds generate a lower real return which averages about 2.5% over the whole sample. The equally weighted average of the three asset classes is close to 6%. Piazzesi & Schneider (2016) show data for the ratio of US house prices to rents that averages about 13 over 1960-2015. The implied yield of about 8% exceeds the real return by depreciation of structures and other property expenses, but real capital gains need to be added; the net figure is probably a bit under 8%. A figure of 6-7% seems reasonable for the past average return on real assets. If we assume the steady state real rate of return is around 6.5% then based on the values for γ, g, m above (respectively 1.5, 0.02 and 0.01) the implied value of θ is around 0.02. That is the value we take for the rate of time preference.

$\boldsymbol{\alpha}$: We set α (the share parameter in the production function) to the typical share of capital in private domestic value added in developed economies in recent years. This figure is around 0.3, (Rognlie 2016).

$\boldsymbol{\varepsilon}$: Muth (1971) estimates the elasticity of substitution between land and structures in producing housing at 0.5; later work finds a slightly higher level, but well under 1. Thorsnes (1997) puts estimates in the range 0.5 to 1. Ahlfeldt & McMillen (2014) suggest it might be a bit under 1. Kiyotaki et al. (2011) constrain it to 1 in their calibrated model. But the weight of evidence is for a number under 1. Combes et al. (2016) use a rich data set on French housing and estimate a figure of around 0.8 for the substitutability between land

and structures (they quote a central estimate of 0.795 when they use instrumental variables estimation). For our base case we use a value of 0.5. We also consider higher values.

ρ : There are many estimates from the empirical literature on housing of the elasticity of substitution between housing and consumption in utility. Ermisch et al. (1996) summarizes that literature and puts the absolute value at between 0.5 and 0.8; Rognlie (2016) uses a range of 0.4 to 0.8. Kiyotaki et al. (2011) constrain it to 1 in their calibrated model. Van Nieuwerburgh & Weill (2010) use 0.5 for the price elasticity of demand for housing, basing their choice on micro studies. Albouy et al. (2014) and Albouy et al. (2016) find strong US evidence for a value of 2/3. For our base case we use a value of 0.6; we also consider higher values.

The two constants in the production functions for goods and for housing (A, A_s) simply define units and have no impact on ratios and can be set to unity.

There are two share parameters that, in conjunction with other parameters set above, we set to try to match key features of the data. These parameters are:

a: the weight on consumer goods (relative to housing consumption) in utility,

b: the weight of buildings in production of housing.

We chose parameter values, $a = 0.85$ and $b = 0.78$, which generate simulation results that in the steady state version of the model (which we believe relevant to many economies in the period from the mid-nineteenth to the mid-twentieth century) match key US aggregate ratios. Specifically we aim to match key US ratios for the decade 1950-1960 which is near the end of the period where we believe transport improvements were substantial enough to satisfy the balanced growth condition.

In assuming that λ falls by 1.5% a year between 1870 and 1945 we have set the improvement in transport efficiency between the mid-nineteenth and mid-twentieth century equal to one half the sum of the rate of productivity growth (assumed to be 2%) and population growth (assumed to be 1%). Those productivity and population figures are plausible for the US. It is also plausible that in the land-rich US there remains undeveloped land suitable for housing at the periphery of many existing residential areas. This is a condition for balanced growth - we must not have reached the "edge" of the country. (It is a condition less likely to hold for many countries in (relatively crowded) Europe and in Japan - a point we shall return to below). So it makes sense to judge the plausibility of the overall calibration by whether it generates ratios over a balanced growth period that are in line with US aggregate ratios from the mid-twentieth century. Given this criterion we set the initial level of transport costs ($\lambda_{1870} = 0.36$ - the final parameter to be set) to generate a level of land values relative to GDP along a balanced growth path that roughly matches the US value of residential land to GDP over the decade 1950-60.

	US Average 1950-60	Ratios along Balanced Path
Return on Capital	6.5%	6.5%
Housing / Total Consumption	13.8%	14.1%
Total Consumption / GDP	77%	85%
Gross Profit Share	35.4%	37.8%
Net Profit Share	26.8%	26.2%
Buidings (B) / GDP	102%	96%
Residential Land to GDP	82%	87%
Capital (K) / GDP	180%	197%
Total Wealth / GDP	364%	381%
Total Wealth / NDP	413%	452%

Table 1: Comparison of Model generated and observed Target Ratios.

Source: US Bureau of Economic Analysis; Board of Governors of the Federal Reserve System. Return on capital is profits net of depreciation plus consumption of housing services relative to the sum of the value of residential and non-residential capital. Buildings are values of residential fixed capital. Residential land is estimated by subtracting the value of residential buildings from the reported value of residential real estate. Total wealth is the sum of the values of land, residential structures and non residential fixed assets.

4 Results

4.1 Balanced growth (1870-1950)

Table 1 shows what the model generates for key ratios on a balanced growth path and compares them with ratios for the US averaged over the period 1950-60.

The table shows that the balanced growth path of the calibrated model matches the US data from the mid-twentieth century quite closely - something that is not guaranteed because the number of free parameters is less than the ratios we are trying to match.

On the balanced growth path, when land is available to build on at the expanding periphery of urban centers that transport improvements made feasible, then house prices and rents are steady. On that balanced growth path urban areas should grow at the rate of their urban GDP. The model predicts that slowing transport improvements would mean that the ratio of urban areas to their GDP would start to fall. We consider whether this feature of the model is in accord with the history of Europe's two largest (and amongst its oldest) cities: Paris and London. Table 2 uses data on the size of Paris and London and an estimate of those cities' GDPs over time. The GDP estimates are crude: population of the cities multiplied by average GDP per capita for the country. Effectively we assume a constant ratio of per capita incomes in the capital to the rest of the country. The built up urban areas of the cities over the past 220 years are more reliable. The table shows an enormous expansion of the urban footprint for these two cities over time. Population density within each city declines steadily over time. Between 1800 and 2014 the urban area of Paris is estimated to have increased by a factor of over 200; for London it has risen by a factor of around 70. The ratio of urban area to urban GDP is, relative to that enormous

	City Pop. (hectares)	Urban Area	Density (Pers. / Hect.)	GDP per Capita (1990 $)	Hectares / City GDP		City Pop. (hectares)	Urban Area	Density (Pers. / Hect.)	GDP per Capita (1990 $)	Hectares / City GDP
Year	(Mn)	'000s		'000s		Year	(Mn)	'000s		'000s	
			Paris						London		
1800	0.58	1.2	497	1.09	1.84	**1800**	0.96	3.6	265	1.62	2.33
1832	0.89	1.8	488	1.31	1.56	**1830**	1.66	5.1	327	1.75	1.75
						1845	2.10	7.8	272	2.07	1.78
1855	1.16	3.4	339	1.62	1.82	**1860**	2.76	10.8	256	2.83	1.38
1880	2.27	7.8	292	2.12	1.62	**1880**	3.76	16.2	233	3.48	1.24
1900	2.71	22.0	123	2.88	2.82	**1914**	7.32	37.2	197	4.93	1.03
1928	2.88	28.7	100	4.43	2.25	**1929**	8.05	68.0	118	5.50	1.53
1955	6.28	63.8	98	6.20	1.64	**1954**	8.29	107.5	77	7.62	1.70
1979	8.79	93.4	94	14.63	0.73	**1978**	7.63	161.3	47	12.83	1.65
1987	9.28	164.2	57	16.16	1.10	**1989**	9.93	202.2	49	16.41	1.24
2000	10.01	231.4	43	20.42	1.13	**2000**	9.74	243.0	40	20.35	1.23
2014	11.11	277.8	40	22.22	1.12	**2013**	11.20	250.8	45	23.74	0.94

Table 2: Urbanised Area (Hectares) divided by City GDP ($Mn)

Source: Atlas of Urban Expansion; Maddison Project Database, version 2018 The figures for Urban Area and Density after 2000 refer to those published under the title 'Total Urban Extent' in Angel et al. (2016). Those prior to 2000 were provided separately by the authors and refer also to 'Total Urban Extent'. GDP per capita figures are from Bolt et al. (2018) and are in 1990 International Geary-Khamis dollars.

variability, dramatically stable. In Paris and in London that ratio was about the same in 1950 as it had been in 1850. But since around 1950 the urban area to GDP ratio has fallen by around 45% in London and by around 35% in Paris. That is what the model predicts will happen once we are off the balanced growth path because transport improvements slow down.

We stress that there is nothing in the model to put the economy on a balanced growth path and keep it there. That productivity improvements in many developed countries saw travel speeds rise roughly in line with the average of population and labor productivity growth between the mid nineteenth and mid twentieth centuries may have made it an unusual 100 years. There is little to suggest it was true before then and much evidence that is has not been true in the post second world war period.

4.2 Off the balanced growth path (the post 1950 world)

We now focus on what the model implies will happen outside of the period when the conditions for balanced growth are satisfied. We begin by considering what happens when improvements in transport speeds slow down and fall short of half the sum of labour productivity plus population growth - which our evidence on transport speeds suggests began to happen around the middle of the twentieth century in many developed economies and which has been followed in recent decades by near stagnation in travel speeds from around the mid 1980s.

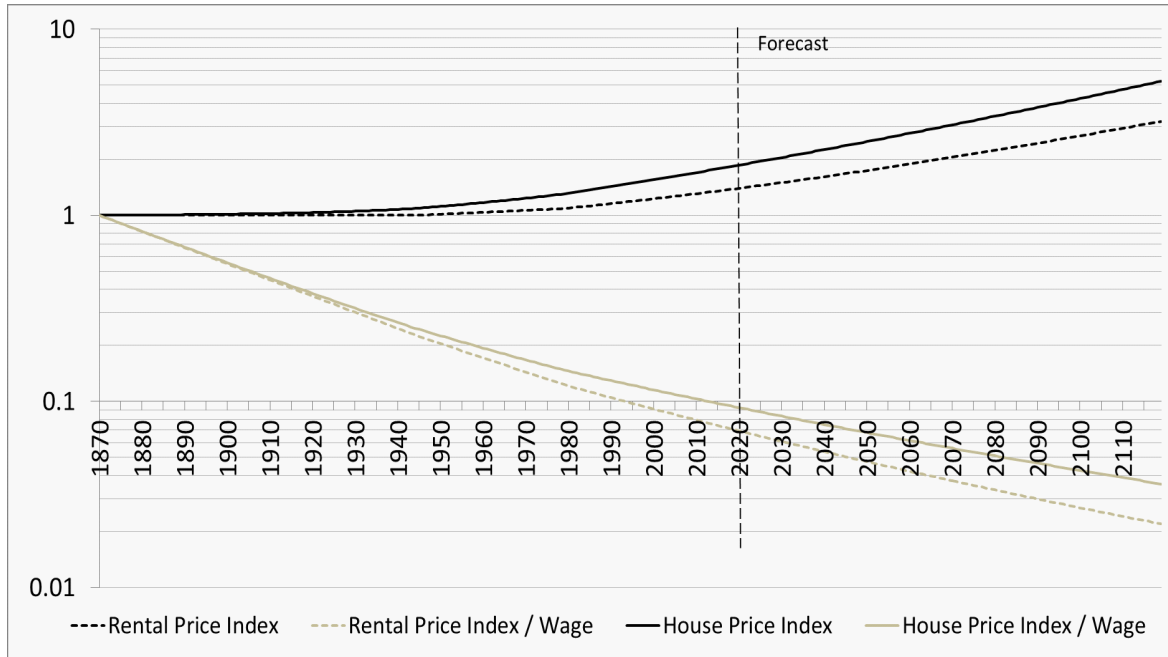We illustrate the effects by focusing on a period of 250 years. We think of this period

Figure 1: Path of the rental price and the rental price relative to wage.

as running from towards the end of the nineteenth century to 100 years ahead - from around 1870 to 2120. We chose the first 150 years of this period to roughly correspond to the period over which Knoll et al. (2017) measure house and land prices. So we have set population growth rates and the trajectory of travel costs over the first 150 years to reflect our assessment of their broad shape over the period since 1870. We noted above that the estimates from Knoll et al. (2017) averaged across all of the (now rich) countries they analyze show the real price of housing roughly flat for almost 100 years and then almost tripling since about 1950. But there is diversity across countries in the path of real house prices since the end of the second world war - real price increases have tended to be significantly higher in Europe and in Japan than in the US. A key factor here is likely to be the very different amounts of land relative to population. But first we focus just on changing patterns of transport improvements so retain the assumption that residential development does not reach the edge of the country (or get stopped by planning restrictions) over the whole horizon.

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Figure 1 shows the path for the real cost of housing for this simulation. This is the rental equivalent (averaged across locations) and labelled "rental price index", p_{lt}. The figure also shows an index of real house prices, p_t^{House} defined in equation (30). The figures here are population weighted averages across locations at each point in time normalized to 1 in 1870. For this base case we set $\varepsilon = 0.5$ and $\rho = 0.6$. There is very little change in the real cost of housing (rents) between 1870 and the middle of the twentieth century. The same is true for real house prices. Between around 1945 and 2020 the user cost of housing (rental price
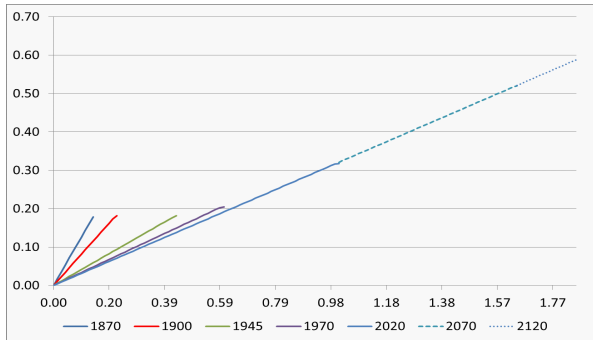
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index)) rises by almost 40%. The average house price rises by considerably more - the level in 2020 is about 1.7 times as great as in 1945. Based on the findings of Knoll et al. (2017) this is significantly lower than the average increase for developed countries on the whole - where average prices trebled - but about right for the US.

Over the next 100 years, and assuming no further improvement in transport costs, both the user cost of housing and average house prices are forecast by the model to rise consistently and at a rate which is gently accelerating. Between 2020 and 2070 the real user cost of housing (or rents) rise by 50% and real house prices rise by about 65%. Over the fifty years after that rents go up by about 60% and house prices by just over 70%. But housing costs (both rents and house prices) do not rise faster than incomes. As we shall see shortly this picture is very different if we use a much lower ratio of land to population and one which means that countries have already developed (even at low density) much of their land for housing. The real interest rate declines very gradually after transport improvement stop because the growth in consumption is slightly slower (see equation (42)).

The spatial pattern of development, the density of population across regions and the evolution of transport costs are shown in the panels of figure 2. Here we show the dispersion of outcomes across locations at intervals of several decades. The horizontal axis measures the distance from the center as a proportion of the distance to where the periphery of development is projected to be at 2020. As transport costs decline rapidly through the late nineteenth century and up until about the middle of the twentieth century people are able to commute greater distances and it becomes feasible to live further from the center. The model implies that the average distance that people live from the center (which in the stylized model one would expect is linked to the average commute distance) roughly doubles between 1870 and about 1920. The limit of commercially feasible residential development over this period also roughly doubles. The average distance people live from the center is predicted to continue rising even though transport improvements stop. Travel costs become a rising proportion of total spending.

For the household that lives the average distance from the center at the end of the nineteenth century the cost of living at that location is the equivalent of facing prices for consumer goods higher than at the center by about 12%. So with consumption of goods around 86% of spending (the rest is spending on housing) this is roughly the equivalent of income being around 10% lower. In terms of time that would roughly be worth 10% of the average work week. By 2020 the transport tax was a little higher - rising to the equivalent of close to 17% on the price of consumer goods. Consumption of goods was predicted to be around 84% of all spending by then so that the cost of distance could be seen as the time equivalent of around 14% of average hours worked. That would be a daily commute time of close to 1 hour for an 8 hour work day, or an average commute time per trip to work of 30 minutes - a plausible figure for average travel time to work in major cities. The typical travel cost varies rather little over time - with average distance travelled rising roughly in line with increasing travel speeds up until transport technology stops improving.
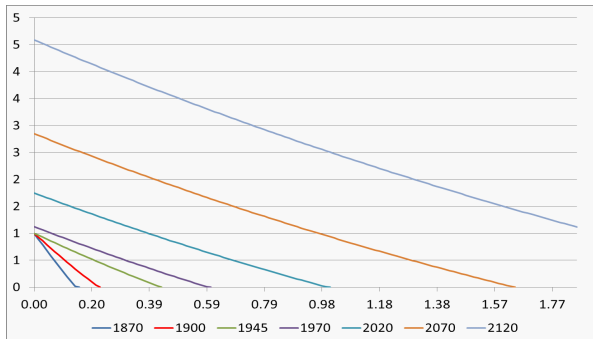
Panel A: Transport Costs over time   Panel B: Rental Prices
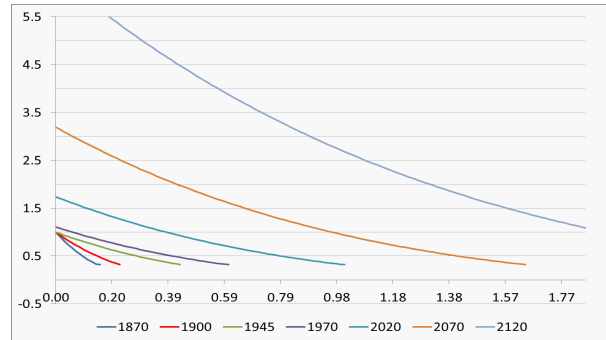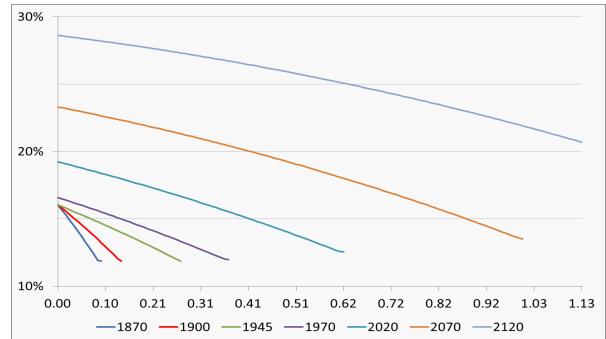

Panel C: Structure to Land Ratio   Panel D: % Expenditure on Housing
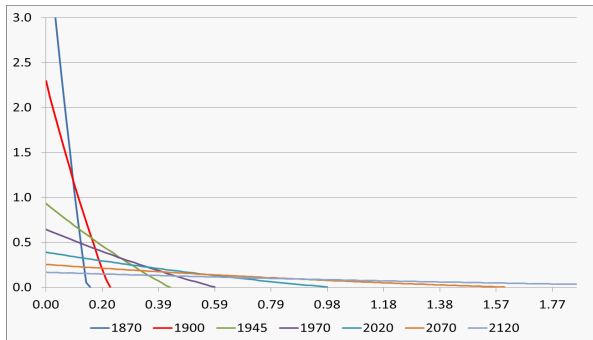

Panel E: Dynastic Population Density

Figure 2: The Distribution of Activity across Time and Location.

Note: In all panels the x-axis is distance from the CBD, normalized by the radius of periphary of urban development in 2020. Panel A plots the transports costs as a % of consumption. Panel B plots the rental rates normalized so that the rental price at the center in 1870 is 1. Panel C plots the structures to land ratios normalized so that ratio at the center in 1870 is 1. Panel D is the % of household expenditure on housing. Panel E is the dynastic population density.

Prices are always highest at the center but one implication of improving transport over the past 150 years is that the ratio of housing costs further from the center to those at the center rises. The proportion of the country which is developed for housing also rises. In 1870 only land within a circle whose radius was about 14% of the distance to the edge of development by 2020 was used for housing; by 1920 land one third of the way to the edge of 2020 developments had houses on it and by 1970 it was 60% of the way there. By about 2070 a combination of rising population, growing incomes and constant travel costs means that the distance from the centre that is developed is double its 2020 level. Density of population is always the greatest at the center, but population density is forecast to become much more equal over the next decades; density at the center is predicted to have declined continuously since the mid-nineteenth century and to continue to do so.

How do these predictions on the geographical pattern of development, of population densities, on commuting and on the price-distance gradient match up to the historical record?

Two features of the simulations are clear and are obvious implications of the fall in transport costs over the past. These are that average commuting distances are predicted to have risen significantly and that price gradients (the fall in rent as distance from the center rises) have become less steep. Both features are strongly supported by the historical evidence. We noted above the strong upward trajectory of commuting distances over time. The decline in the house price-distance gradient over time has been noted many times in the literature and goes back at least to Mills (1972) who noted evidence of flattening in the gradient in the US which began in the nineteenth century. There is evidence of flattening in land price gradients for numerous cities, including: Chicago (McMillen 1996); Berlin (Ahlfeldt & Wendland 2011); Cleveland (Smith 2003); New York (Atack & Margo 1998); Sydney (Abelson 1997).

Other predictions of the simulations are not so obviously consistent with the historical evidence. The model predicts that the developed areas around cities should have consistently expanded (as has population) but that the density of population at the center will decline and density become more even across the country. This is precisely what has happened in just about all major world cities over the past 25 years. The population of cities in developed countries increased by a factor of 1.2 between 1990 and 2015; their urban extents increased by a factor of 1.8. In developed countries population densities in cities declined at 1.5% a year during this period. (Source: Atlas of Urban Expansion, 2016, vol 1 available at www.atlasofurbanexpansion.org). Table 3 gives details for 12 of the largest cities in the developed world. In every case density has fallen over the past 25 years. .

The model also predicts that commuting distances will tend to rise over time but that expenditure on transport will be roughly constant as a share of total spending.

	1990		2014	
	Urban Area Hectares	Population Density Person/Hectares	Urban Area Hectares	Population Density Person/Hectares
Berlin	24,707	131.5	68,742	56.2
Chicago	340,561	21.5	510,971	17.4
Hong Kong	5,933	617.5	9,253	467.1
Houston	116,357	23.5	272,394	19.8
London	131,464	64.8	177,272	63.2
Los Angeles	353,940	34.9	459,046	33
Madrid	20,632	176.5	56,019	93.8
Milan	51,115	68.6	178,364	35.9
New York	509,235	31.9	747,852	24.6
Paris	127,790	72.5	198,625	56
Sydney	69,122	41.5	110,033	37.4
Tokyo	278,694	104.7	448,929	77.4

Table 3: Table recording both Area and Population Density in 1990 and 2014 for selected cities.

Source: Atlas of Urban Expansion, (Angel et al. 2016); The figures refer to Built-Up Area, and Built-Up Area Density for the respective cities.

4.3   Running out of land to develop

The simulation on the past and future trajectory of housing described above and illustrated in Figure 1 are based on a calibration of the model that does a reasonable job at matching aggregate characteristics for the US. But it gets the housing market wrong for many other developed countries. In particular real house prices have risen far more strongly over the past 50 years in much of Europe and in Japan than in the US. One reason why this might be true is that the population density of the US is much lower than in most European countries and very much lower than in Japan. Land area relative to population in the US at around 2015 was about 7 times as great as in Germany, just over 8 times as great as in the U.K. and around 10 times as great as in Japan. Areas that can be developed around many (though certainly not all) US cities are more plentiful than is the case in much of Europe and in Japan. We ran the model with the same calibration as above (with slowing transport improvements after 1945) except that the area of the economy relative to population is set to be much lower. We now show simulation results for a version of the model where the available land area of the economy is such that by 1990 there is for the first time some (very low density) residential development at the edge of the country. One might think of this as a simulation to reflect conditions in "crowded old Europe" in contrast to "land rich US". But another interpretation is that restrictions on residential development around cities began to bind after 1990. Figure 3 shows the predicted evolution of rents and house prices under this assumption of much more limited available land for new houses.

Several things stand out from the figure. First, in the period when there was still unused land to develop *and* transport costs were coming down (so that the conditions for balanced
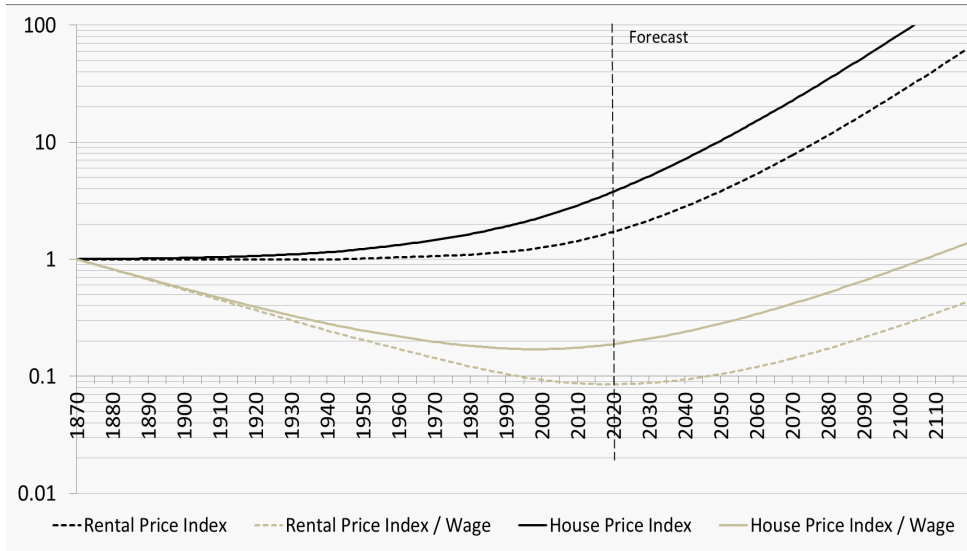
Figure 3: Running out of undeveloped land: Path of the rental and house price indices - absolute and relative to wages

growth were roughly satisfied) real house prices and rents were flat and both fell relative to incomes. That period is predicted to have ended in the middle of the twentieth century. In the period since the end of the second world war until 2020 both rents and real house prices are forecast to have risen far more than in the previous simulation where undeveloped land is always available and only the slowing of transport improvements took us away from the balanced growth path: rents rise by about 70% and house prices by 2020 have more than tripled relative to the level in 1945 when we have both the petering out of transport improvements *and* the ending of freely available undeveloped land. Over the next 50 years (2020 to 2070) figure 3 shows real housing costs (rents) are projected to rise fourfold and in the 50 years after that (between about 2070 and 2120) to rise eightfold. House prices rise by much more: prices rise six-fold between 2020 and 2070 and between 2070 and 2120 they rise nine-fold. Unlike in the simulations where we only have a slowdown in travel improvements (but still undeveloped land is available), both rents and house prices are projected to now rise consistently faster than average incomes.

The key factor here is that with completely undeveloped land assumed to be exhausted, with relatively low substitutability between land and structure (0.5), less than unit elasticity of substitution between housing and goods (0.6) and with no assumed further improvement in transport costs then housing costs rise at much faster rates than in the past. The scale of this effect, as we shall see, is highly sensitive to those factors.

This difference in the trajectory of land and house prices between the land-rich and land-poor simulations does not only affect house values, the price-distance gradient and the distribution of population densities - it also affects the incentives to invest in productive capital, the capital to output ratio and the rate of return (the interest rate). For the land-

rich simulations real returns on capital stays roughly constant at around 6.5%. For the land-poor simulations the rate of return from 2020 steadily rises towards 7%. This reflects a crowding out of investment in productive capital by more investment in structures as the fast rising land price encourages builders to switch the mix of land and structure towards the latter.

4.4   Zoning Restrictions and welfare

Restrictions on the ability to build residential structures at various locations have played no role thus far. But there is a good deal of evidence that such restrictions do play a role; they might even be a much more significant factor in explaining rising house prices in recent decades than a slowing in transport improvements and more significant than countries running out of undeveloped land (see for example Quigley & Raphael (2005), Glaeser et al. (2005b), Glaeser et al. (2005a), Jaccard (2011)).

We explore this issue by introducing restrictions on building into our model. Specifically, we now allow for restrictions on the amount of residential structures that can be erected at specific locations. We impose a limit on the ratio of residential building to land area at different locations – that is an upper limit on B_l/R_l.

We consider uniform limits on the B_l/R_l that apply at all locations. We also consider an alternative – more realistic - restriction that from a given date (which we take to be 1970) then at all locations beyond a certain distance $v = 0.2$[7] from the center the building density cannot rise beyond the 1970 value for that density at distance $v = 0.2$. Thus for all $l > v$, $B_l/R_l \leq (B_v/R_v)_{1970}$. We freeze all the building densities in the area closer than $v = 0.2$ to the center at their 1970 values. This second set of restrictions is a form of zoning that allows existing structures within the v radius to continue standing (but not to be added to) and puts a limit at all points beyond distance $v = 0.2$ that development can never be more intense than it was at that location in 1970.

Tables 4 and 5 show the results. For the fixed limit on B/R we consider limits at 2 and 3. For the zoning restriction introduced in 1970 we consider limits that are tighter again. We focus on those results from the tighter restrictions to gauge how important zoning restrictions can be.

Without zoning restrictions and no shortage of land the only reason for land and house prices to rise is due to slowing transport improvements. The tables show that if that was all that happened the impact on welfare and house prices (relative to the balanced growth path) are very substantial.

Focusing first on welfare, we calculate the compensating variation in incomes (or expenditure) that would leave utility unchanged when transport improvements slow down relative to a world where there was no slowdown so the economy remained on a balanced growth path. The ending of transport improvements we assume began in 1980 and by 2020 means that incomes would have need to be almost 8% higher. By 2050 that becomes over 14%

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[7]Distance, l, was normalized so that in 2020 the urban developed area was of unit size.

	Required Compensating Expenditure						Average House Price					
	Diminishing Growth in Transport Speeds						Balanced	Diminishing Growth in Transport Speeds				
			and				Growth			and		
Year	Only	Running out of Land	$B_l/R_l \leq 3$	$B_l/R_l \leq 2$	$B_l/R_l \leq (B/R)_{1970}$			Only	Running out of Land	$B_l/R_l \leq 3$	$B_l/R_l \leq 2$	$B_l/R_l \leq (B/R)_{1970}$
1950	0.2%	0.2%	0.2%	0.2%	0.2%		1.00	1.12	1.23	1.13	1.14	1.17
1980	1.8%	1.8%	1.8%	1.9%	1.7%		1.00	1.31	1.64	1.33	1.37	1.44
2000	4.5%	4.5%	4.5%	4.7%	4.9%		1.00	1.55	2.28	1.59	1.63	1.74
2020	7.9%	8.2%	7.9%	8.3%	9.0%		1.00	1.86	3.72	1.92	1.98	2.11
2050	14.3%	18.0%	14.6%	15.7%	17.4%		1.00	2.48	10.06	2.60	2.69	2.89
2070	19.6%	29.4%	20.4%	22.1%	24.7%		1.00	3.05	22.05	3.21	3.34	3.60

Table 4: Impact of house building constraints on welfare and house prices; all parameters set to base values

Note: The left table records the compensating expenditures with respect to the balanced growth path (% increase in household exenditure necessary for households to acheive the same utility as on the balanced growth path). For all other columns the growth rate in transport speeds fall after 1945 from 1.5% to 0.8% and then 0% after 1980. For the first column this is the only difference. The other columns have an additional constraint. In the second column, the urban area hits the 'edge' in 1990; in the third the structure to land ratio is constrained to be 3 or below; in the fourth this is constrained to be 2 or below; and in the fifth this ratio is constrained to be on or below its level in 1970 out to a radius of 0.2 and no higher than this value further out. The right table records the average house price. Here there six columns. The first records the average house price along the balanced growth path (which is constant and normalised to 1) and the other five columns give the average house price with respect to the five scenarios described in the left table.

	Required Compensating Expenditure						Average House Price					
	Diminishing Growth in Transport Speeds						Balanced	Diminishing Growth in Transport Speeds				
			and				Growth			and		
Year	Only	Running out of Land	$B_l/R_l \leq 3$	$B_l/R_l \leq 2$	$B_l/R_l \leq (B/R)_{1970}$			Only	Running out of Land	$B_l/R_l \leq 3$	$B_l/R_l \leq 2$	$B_l/R_l \leq (B/R)_{1970}$
1950	0.2%	0.2%	0.3%	0.5%	0.2%		1.00	1.08	1.10	1.12	1.15	1.18
1980	1.7%	1.7%	1.7%	2.1%	1.5%		1.00	1.23	1.29	1.33	1.38	1.50
2000	3.9%	4.0%	4.2%	4.8%	4.7%		1.00	1.43	1.56	1.59	1.65	1.81
2020	6.6%	6.7%	7.4%	8.5%	8.9%		1.00	1.67	1.97	1.91	2.00	2.20
2050	11.3%	11.9%	13.9%	15.8%	17.4%		1.00	2.14	3.14	2.57	2.72	3.00
2070	15.2%	16.5%	19.6%	22.2%	24.8%		1.00	2.56	4.60	3.17	3.38	3.74

Table 5: Impact of house building constraints on welfare and house prices; parameters set to base values, except $\epsilon = 0.75$.

Note: The left table records the compensating expenditures with respect to the balanced growth path; the right table records the average house price. The scenarios are identical to those in Table 4 and are described in the footnote to that table.

and by 2070 the compensating variation is close to 20%. Adding in zoning restrictions that start in 1970, and which by 2020 are binding across much of the economy, does not reduce welfare further by a large magnitude. The compensating income effects rise from just under 8% to 9% in 2020, from 14.3% to 17.4% in 2050 and from 20% to 24.7% in 2070. So the welfare effects of zoning are ultimately around one quarter of the effects of slowing transport improvements. This is not because the building restrictions are hardly binding. Without restriction building density (B/R) near the central location in 2070 would be almost three times as great.

The impact on house prices reveals something similar: zoning restrictions are much less powerful than are the effects of the assumed slowdown in transport improvements. In 2020 slowing travel improvements raise average house prices relative to balanced growth by over 80%; by 2050 prices are higher by a factor of 2.5 and by 2070 by a factor of just over 3. The impact of zoning restrictions on top of that is by no means trivial – by 2070 prices would be up by a factor of 3.6 rather than just over 3. But the effects of zoning shows up as consistently being roughly one fifth that of slowing transport improvements.

The effects of restrictions on building residential structures are greater if land and structures are more substitutable: putting limits on structure is more onerous if, as transport improvements fall, you would want to substitute more towards structure and economize on land. Table 5 shows this. Here we assume substitutability between land and structure, ε, is 0.75 rather than the base case of 0.5. The zoning effect is about 50% as great as the travel effect; that effect was only around 20% as great as the travel effect when ε was 0.5.

4.5   Sensitivity to key elasticities

For the base parameters rents and house prices are predicted to rise dramatically over coming decades in land-poor countries which until a few decades ago had experienced flat prices. We now assess the sensitivity of that projection of future housing costs to three factors: 1. substitutability between land and structure in creating housing; 2 substitutability of housing for consumption goods in utility; 3 variations in the path for transport costs (speed of commuting). In each case we focus on what changes mean for the cost of housing relative to consumer goods and the cost of housing relative to incomes. We use an assumption of land availability relative to population in line with what we have called land-poor simulations - that is where completely undeveloped land is no longer available after 1995. Thus the results below are directly comparable with those shown in figure 3 above.

1. $\varepsilon = 0.75$

When $\varepsilon = 0.75$ the path of housing costs over the future looks very different, see figure 4. There is some very gentle rise in real housing costs over the next 100 years, but at a rate significantly below the growth in average earnings. So instead of housing costs (rental equivalents) over the next 50 years rising nearly 60% relative to average labor incomes (when $\varepsilon = 0.5$) costs *fall* by about 30% relative to average earnings. Over that 50 year period house prices fall relative to incomes by about 15% when $\varepsilon = 0.75$ rather than rising strongly

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faster than incomes when $\varepsilon = 0.5$. If $\varepsilon = 0.99$ rental costs and house prices barely rise any faster than the cost of consumer goods and they decline markedly relative to earnings to less than half their 2020 value by 2070. Clearly there is enormous sensitivity in the path of housing costs and of house prices to even relatively small changes in the substitutability between land and structure in creating homes. That degree of substitutability is partly a matter of technology and partly a matter of preferences, reflecting the fact that ε effectively plays a dual role as summarizing production possibilities but also trade-offs in creating utility by consuming different combinations of structure and land (or building and space). This point was recognized by McDonald (1981). He noted that a model with utility a function of consumption of goods and housing, then if housing is "produced" via a CES function combining land and structures, it can be interpreted as a weakly separable utility function rather than a model of the production of housing. This implies that estimates of the elasticity of substitution, ε , can be thought of as estimates of a production parameter or as an estimate of a parameter of a weakly separable utility function. The production function of housing can as well be thought of as an aggregator reflecting tastes. Both tastes and production technology can change and so one should not think of ε as fixed. The degree to which one can substitute structure for land has probably changed significantly over time - new building techniques now make it possible to build 100 story apartment blocks with very small footprints. That creates a different trade-off between use of land and structure in creating units of housing than was available when the Empire State building was constructed - a building that had a footprint that was enormously larger relative to its height than the sort of pencil thin apartment blocks recently constructed in Manhattan. But whether having a living space a mile up in the air - potentially shrouded in cloud for some of the time and remote from life on the ground - is really a good substitute for a second floor apartment is not a question of technology. So while ε may rise with technological improvements - and put some limit to the rise in housing costs in the face of fixed land area - that scope is strictly limited by preferences which might be completely insensitive to changing housing costs.

2. $\rho = 0.90$

A moderately higher degree of substitutability between housing and consumer goods (0.9 versus 0.6 in the base case) has a very substantial effect on the future evolution of housing costs, see figure 5. Over a 50 year horizon the impact is almost as great as varying the elasticity of substitution between land and structures - it is the difference between rents rising steadily relative to incomes - ending up over 65% higher by 2070 - or rising by only just over 10% faster than incomes.

3. Transport costs:

In the base case we assume that improvements in travel speed stopped in 1980 and do not change over the next 125 years. If instead travel times from 2020 once again start to fall at 0.8% a year - the same rate as assumed between 1945 and 1980 - there is, not surprisingly, predicted to be a somewhat more equal density of population over the country. There is
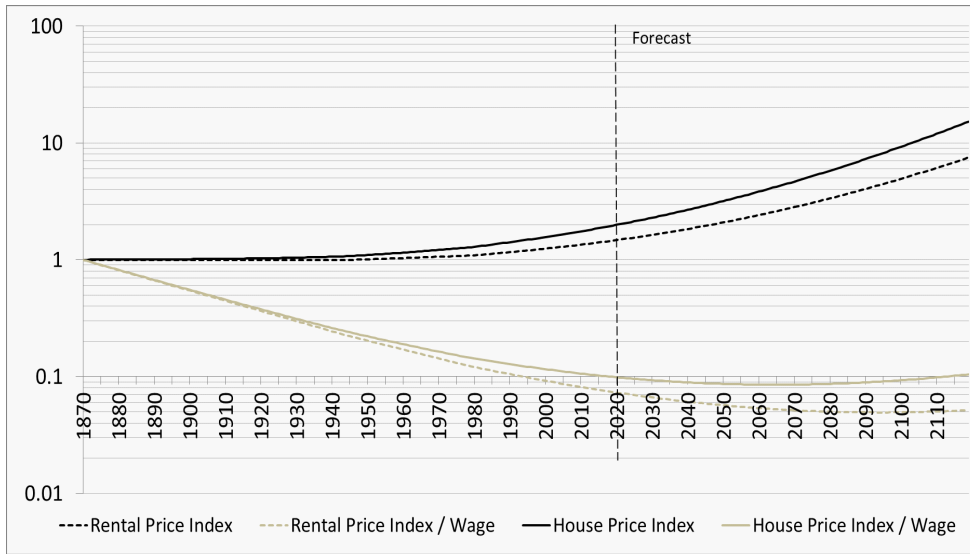
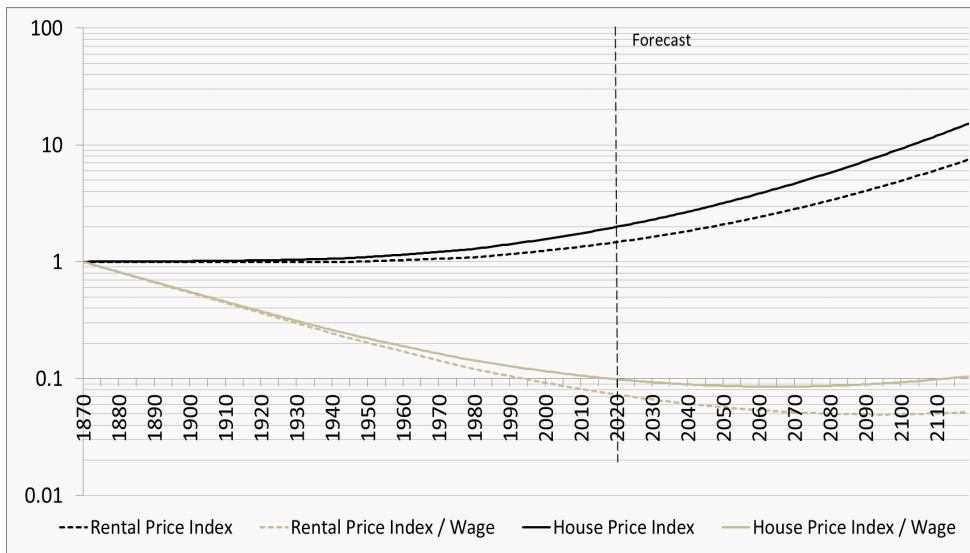Figure 4: Path of the rental and house price indices - $\varepsilon = 0.75$

Figure 5: Path of the rental and house price indices - $\rho = 0.9$

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also greater investment in structures and overall consumption of housing is greater. For the 'land-poor' simulations the effects are not enormous - even if we go back to transport efficiency rising at late nineteenth century levels. But for 'land-rich' simulations if transport improvements were to resume the impacts are greater. If such improvements once again occurred at the rate typical in the late nineteenth century - so that the conditions for balanced growth once again held - house prices and rents would once again be flat and so fall sharply over time relative to incomes.

The 'land-poor' simulations reveal that the impact of further falls in transport costs on the overall level of housing costs over the future is far smaller than its impact in the past. We ran a simulation assuming no transport improvements at all between 1870 and 1970; the level of real housing costs in 1970 was twice as high as in the base case which calibrated travel improvements to evidence on speed of moving passengers. Yet assuming continuing improvements in travel speeds from now on had a small effect on house prices 100 years ahead in land-poor countries relative to the assumption of no further changes in passenger speeds. Why do transport cost improvements have a relatively small impact upon the trajectory of average housing costs in the future and yet have been very significant over the past? The main reason is that there are diminishing returns to travel improvements. In our model the travel improvements between 1870 and 1920 substantially increase the proportion of the economy where it is viable to live. By 1990 (for the 'land-poor' simulations using European-Japanese population to land ratios) almost all the country has some residential development, although density is still very much lower nearer the periphery than at the center. Until this period there had effectively been a major expansion in the physical size of the country that is relevant for the day to day lives of the population. After that date further improvements have some impact on density differences between areas that have already been developed, but they do not increase the area that has some housing on it.

4.6   Caveats and Limitations:

In assuming that dynasties are long-lived and are alike (except for where they chose to live) we give no useful role to mortgages. Essentially we have representative agents so there are not distinct groups alive at the same time some of whom want to save and some of whom want to borrow. Obviously such a model has no meaningful role for mortgages or indeed any debt. It is not that holding debt and financial assets could not be envisaged in the model - we could certainly allow households to have mortgages and save in financial assets that both pay the same interest rate. But in a closed-economy model with representative households the only equilibrium would be one where each household held an amount of interest bearing assets equal to its interest paying debt. Neither does the model make any meaningful distinction between owner occupation and renting; the cost of either form of tenure is the user cost of housing. So the model does not tie down the owner-occupation rate. We could interpret the outcomes as being ones where some households rent and simultaneously own shares in property owning companies that rent out property and pay

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the returns to shareholders. Ultimately the members of dynasties alive at any time own all the housing stock - and it does not matter whether it is held as owner occupied property or as claims on properties that are rented out. Members of those dynasties pass housing wealth down to later members of the same dynasty. This does reflect an important aspect of reality: people do pass on lots of wealth (both at death and in life) and housing is, for most families, the largest part of bequests. But we completely miss out on issues of inequality and the impact of credit restrictions by making assumptions that dynasties are long-lived and are all alike except for where they live.

We have considered the effects of planning and zoning restrictions - but have done so in a particular way that may not capture the full extent of their impact. But our finding that travel costs are in the long run more important than building restrictions is one we think plausible. There is a sense in which transport technology is ultimately a more powerful and fundamental force than planning restrictions in and around cities. If people could safely and reliably travel 200 miles to the center of a city in 30 minutes then planning restrictions on residential developments in a 10 mile radius about the center of that city would not be very important. To take just one example. London has had a green belt around it for many decades and that has restricted building new houses. But even with such a green belt, had transport improvements continued at the pace of the nineteenth and early twentieth centuries it would have allowed the population that could work in London to expand as it became feasible to live ever further beyond the green belt in areas where residential development was allowed.

5   Conclusions

This paper develops a model of the changes, over time and across locations, in housing and of housing costs. We find that so long as improvements in travel technology proceed at a pace that is in a fixed proportion (of one half) to the growth in productive potential (the sum of labor force growth and general productivity growth) there is a balanced growth path with no change in real house prices. But once travel improvements fall back we are no longer on a balanced growth path and real house prices and rents rise. How fast they then rise becomes dependent on a range of parameters that play little role when travel improvements were at a rate near half GDP growth. We then find that plausible parameter estimates plugged into this growth model can easily generate ever rising housing costs – relative to the price of other goods *and* to incomes. But there is great sensitivity of that to parameters that reflect both preferences (between different characteristics of houses) and technology. One key technology factor is how one combines structures and land to create housing. That has changed - the New York skyline shows that it is now possible to erect super-tall residential buildings on small plots of land and squeeze more residential space from a plot than was possible in the past. Whether that can drive our parameter ε to

higher levels that fundamentally change the likely future cost of housing is an interesting question.

Price sensitivity of demand for housing - reflecting the substitutability of housing for other goods and services in creating satisfaction - is another factor with a very powerful effect on the longer term path of housing costs. The great sensitivity of the equilibrium (or fundamental) housing cost trajectory to small changes in two key elasticities means it is hard to know whether house prices relative to incomes rising to levels not seen before is the start of a bubble or just the natural path we should expect in an economy with a rising population, growing incomes and with a population density higher than in the past.

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