Problems with Using CDS to Infer Default Probabilities

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Abstract

Using credit default swaps (CDS) to imply a firm's or sovereign's default probability is laden with difficulties, making the resulting estimate unreliable. This paper exposes these difficulties using a simple analogy to life insurance premiums. An analogy is used because the logic is more easily understood in this context. The difficulties are unraveling the impact of risk premium, counterparty risk, market frictions, and strategic trading. Given a well understood alternative to implied CDS default probabilities is available, actuarial based default probabilities, banking regulations and risk management decisions should not be based on CDS implied default probabilities.

1 Introduction

It is commonly believed that credit default swap (CDS) implied probabilities provide unbiased estimates of a corporation's or sovereign's actual default probabilities or credit risk. The purpose of this paper is to show why this common belief is false using an intuitive analogy between CDS and life insurance premiums. For life insurance, the difficulties of using insurance premiums to imply the mortality probability are crystal clear. The difficulties are unraveling the impact of risk premium, counterparty risk, market frictions, and strategic trading. These difficulties are well known in the academic literature (references are provided below). Of course, all is not lost because there is no need to use these implied probabilities. Readily available actuarial based mortality tables are used to determine mortality probabilities. And insurance premium implied mortality probabilities are not used for risk management nor in regulation.

Drawing an exact analogy between CDS and life insurance, this logic shows that implied CDS default probabilities do not provide reliable estimates of a

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firm's or corporation's default. They are unreliable due to the difficulty in unraveling the impact of risk premium, counterparty risk, market frictions, and strategic trading. As such, their use in risk management and regulation is problematic. As an alternative, actuarial based default probabilities should be used instead. This is an important observation given the recently enacted Dodd-Frank Wall Street Reform and Consumer Protection Act that correctly requires the removal of credit ratings from all U.S. federal agency regulations, and the newly released proposed rules for risk-based capital standards [14] that discuss the use of CDS spreads for this purpose.

An outline for this paper is as follows. The next section presents the life insurance premium model in a frictionless, competitive, and arbitrage-free economy. Sections 3 and 4 study life insurance companies that are always solvent and insolvent with a positive probability, respectively. Section 5 adds market frictions to the previous model. The reasons for using implied mortality probabilities are discussed in section 6, while section 7 concludes the paper. The analogy between life insurance and CDS is contained in remarks made throughout the paper. We recommend that the reader does not read the remarks on a first pass through the paper, but only on a second pass after the logic and results are understood in the context of insurance premiums.

2 The Model

Consider a single period model with times 0 and 1. For clarity, one may think of this time period as a year. We consider a simple life insurance company whose balance sheet is given in Figure 1. The company sells a single life insurance policy with a notional of N dollars and a premium equal to a fraction $c \in [0,1]$ of the notional value, paid at time 0. If the insured dies over the time period, the notional of N dollars is paid at time 1. We assume that the insured dies with probability $p > 0$.

To guarantee payment, the life insurance company has E_0 dollars of equity. We assume that the company invests both the premiums received and the equity in a default-free money market account. We let the default-free spot rate of interest over $[0,1]$ be denoted $r \in [0,1]$.

We assume that the equity trades in a frictionless and competitive market. By frictionless we mean that there are no transaction costs (bid/ask spreads) and no restrictions on short sales. By competitive we mean that all traders act as price takers, i.e. trades have no quantity impact on prices.

Assets	*Liabilities*
$cN + E_0$ units of the money market account	Life Insurance Policy (premium c, notional N) Equity (E_0)

Figure 1: The Life Insurance Company's Time 0 Balance Sheet.

We study two cases. One is where the life insurance company's equity is sufficient to pay off the life insurance policy at time 1, and one where it is not. Since the life insurance company is of limited liability, in the first case the life insurance company is solvent whether or not the insured person dies. In the second case, if the insured dies, the life insurance company becomes insolvent and the life insurance policy is only partially paid. The two cases are separated by the condition:

$$[E_0 + cN]\,(1 + r) \geq N. \tag{1}$$

The left side of this expression represents the value of the life insurance company's assets at time 1. The solvency condition states that these assets are sufficient to cover the notional value of the life insurance policy if mortality occurs.

In both cases we are interested in: (1) determining the equilibrium life insurance premium at time 0, and (2) implying the mortality probability from the life insurance premium.

For later use, note that one can easily use actuarial mortality tables to estimate the mortality probability. Given this life insurance example, at this point the reader may wonder why one would ever use this insurance premium implied mortality probability instead of the actuarial mortality probability? This is a good question. We withhold an answer to this question until after we determine both the equilibrium insurance premium and the implied mortality probability.

Remark 1 *Here is the analogy:*

(a) the life insurance company is analogous to the seller of a credit default swap (CDS),

(b) the equity is analogous to the CDS seller's collateral,

(c) the life insurance policy is analogous to a CDS, and

(d) the insured person is analogous to the firm underlying the CDS.

The two questions we study are:

(1) determining the market spread of the CDS, and

(2) inferring the default probability of the firm underlying the CDS from the CDS spread.

We note that, analogous to actuarial mortality tables, one can use historical time series of firm defaults to estimate the default probability of the firm underlying the CDS (see Chava and Jarrow [4], Campbell, Hilscher, Szilagyi [2], and Bharath and Shumway [1]).

3 Sufficient Equity Capital

This section answers the two questions posed earlier for a life insurance company with sufficient capital so that it never becomes insolvent, i.e. expression (1) holds.

In this case, the equity's time 1 value is

$$E_1 = \begin{cases} [E_0 + cN](1+r) & if \quad alive \quad with \quad probability\ 1-p \\ [E_0 + cN](1+r) - N & if \quad death \quad with \quad probability \quad p \end{cases}. \tag{2}$$

Remark 2 *The condition of no insolvency is analogous to there being no counterparty risk from the CDS seller.*

3.1 Market Equilibrium

Assuming that the equity market is arbitrage-free, we know that there exists a risk neutral probability $q > 0$ of mortality such that the equity's time 0 value is equal to the risk neutral valuation of its time 1 payout (see Duffie [6]), i.e.

$$E_0 = [E_0 + cN] - \frac{Nq}{1+r}. \tag{3}$$

The actual and the risk neutral probabilities are related by the condition:

$$q = \phi p \tag{4}$$

where $\phi > 0$ is the risk premium.

Given E_0, q, N, and r are known at time 0, equity market equilibrium determines the life insurance policy premium c as the solution to expression (3), i.e.

$$c = \frac{q}{(1+r)}. \tag{5}$$

This premium guarantees that the equity is fairly priced at E_0. Equivalently, this premium sets the life insurance policy's value equal to zero.

Using expression (4), we can rewrite the life insurance premium as

$$c = \frac{\phi p}{(1+r)}. \tag{6}$$

We see here that the equilibrium premium differs from the actual mortality probability by a risk premium. The discount rate appears because the life insurance payout occurs at time 1 and the premium is paid at time 0.

Remark 3 *Expression (2) quantifies the CDS seller's position at time 1 including the CDS premium and collateral. If the firm defaults, the CDS seller's position declines by a payment equal to the notional N. Here the recovery rate is assumed to be zero. One can add a nonzero recovery rate $\delta \in [0,1]$ to this expression by changing the loss $-N$ to $-(1-\delta)N$. In this case the CDS spread is*

$$c = \frac{q(1-\delta)}{(1+r)} \tag{7}$$

where the numerator represents the expected loss. Adjusting for the single period structure, this is similar to the CDS spread obtained in a simple dynamic model, see Jarrow [11].

3.2 The Implied Mortality Probability

Using the life insurance premium c, the implied actual mortality probability is given by

$$p_{implied} = \frac{c(1+r)}{\phi}. \tag{8}$$

The implied mortality probability is a linear function of the insurance premium.

To compute the implied mortality probability an estimate of the mortality risk premium ϕ is needed. Estimating such a risk premium in a non-trivial exercise. It is equivalent to estimating the expected return on the life insurance policy, a procedure which requires an equilibrium asset pricing model including a complete specification of the economy. Financial economists, over 50 years after the initial discovery of the capital asset pricing model, still have not reached a consensus on how to do this (see Cochrane [5]). Indeed, the estimation of risk premium is very difficult. The reason for this difficulty is that the empirical finance literature has documented that risk premium are nonstationary. They vary across time according to both changing tastes and changing economic fundamentals. This nonstationarity makes problematic both the modeling of risk premium and their estimation.

Remark 4 *With a nonzero recovery rate, the implied default probability is*

$$p_{implied} = \frac{c(1+r)}{\phi(1-\delta)}. \tag{9}$$

4 Insufficient Equity Capital

This section answers the two questions posed earlier for a life insurance company with insufficient capital so that in the event of mortality, the life insurance company becomes insolvent. This is strong condition. One could complicate the model further by letting the life insurance company invest in risky assets instead of the money market account, such that if mortality occurs, the company defaults with a probability less than one. In this case, although the resulting formulas will change, the logic and qualitative conclusions will not. Therefore, for simplicity of presentation, we retain this strong condition.

Under this assumption, the equity's time 1 value is

$$E_1 = \begin{cases} [E_0 + cN](1+r) & if \quad alive \quad with \quad probability \ 1-p \\ 0 & if \quad death \quad with \quad probability \quad p \end{cases} . \tag{10}$$

In the event of mortality, the life insurance policy receives a reduced payoff equal to $[E_0 + cN](1+r) > 0$ dollars. This payoff represents the insurance company's time 1 equity.

Remark 5 *If the firm defaults, the CDS seller does make the entire payment required by the contract. Here there is significant counterparty risk in the CDS transaction.*

4.1 Market Equilibrium

Assuming that the equity market is arbitrage-free, we know that there exists a risk neutral probability $q > 0$ such that

$$E_0 = [E_0 + cN](1-q). \tag{11}$$

Again the actual and the risk neutral probabilities are related by the condition

$$q = \phi p \tag{12}$$

where $\phi > 0$ is the risk premium.

Given E_0, q, N, and r are known at time 0, equity market equilibrium determines the life insurance premium c as the solution to expression (11), i.e.

$$c = \left(\frac{E_0}{N}\right)\frac{q}{1-q}. \tag{13}$$

Using expression (12),

$$c = \left(\frac{E_0}{N}\right)\frac{\phi p}{1-\phi p}. \tag{14}$$

The presence of counterparty risk has significantly changed the equilibrium insurance premium. It is no longer linear in the actual mortality probability and it now depends on the life insurance company's equity capital relative to the insurance policy's notional value.

Remark 6 *When there is counterparty risk, the CDS spread is more complex. Note that even if the recovery rate is nonzero, it does not enter the CDS spread because the payment is solely determined by the equity of the CDS seller and the notional value of the contract. The CDS spread is given by expression (14).*

4.2 The Implied Mortality Probability

Using the market premium c, the implied mortality probability is

$$p_{implied} = \frac{c}{\phi}\left(\frac{N}{E_0 + cN}\right). \tag{15}$$

The relation between the insurance premium and the implied mortality probability is much more complex in the presence of insolvency. Note that the implied mortality probability is now a non-linear function of the insurance premium, and it depends on the life insurance company's equity E_0 and the life insurance policy's notional N.

Remark 7 *When there is counterparty risk, the implied default probability is given by expression (15). Here the counterparty's collateral value enters the estimate.*

5 Generalizations

The previous two models were formulated to illustrate the dependence of the implied mortality probability on risk premium and counterparty risk. As shown, these two factors need to be included into an implied mortality probability estimate. However in practice, additional complications need to be taken into account to get unbiased estimates. First, markets are not frictionless, as was assumed in the previous structures. In markets with transaction costs, bid/ask spreads, and short sale restrictions additional modifications need to be made. We consider these modifications next.

First, let's consider the imposition of short sale restrictions. Short sale restrictions on the life insurance company's equity is equivalent to requiring that only the insured can buy life insurance. Jarrow, Protter, and Pulido [13] consider the impact of short sales restrictions on arbitrage-free prices in an otherwise frictionless and competitive market setting. In this case short sale restrictions can lead to equity being overvalued, and the market equilibrium condition changes to

$$E_0 = \begin{cases} [E_0 + cN] - \frac{Nq}{1+r} + \beta & if \quad sufficient\ capital \\ [E_0 + cN]\,(1-q) + \beta & if \quad insufficient\ capital \end{cases}. \tag{16}$$

where $\beta \geq 0$ represents an overpricing of the equity due to short sale restrictions.

Second, let's also add transaction costs and bid/ask spreads, which depend on the quantity traded. Cetin, Jarrow, and Protter [3] show how these frictions can be incorporated into an arbitrage-free economy. These costs are included by letting the time 0 quoted price increase as more shares are purchased, and decrease if more shares are sold, i.e.

$$E_0 + mx \tag{17}$$

where $m > 0$ represents the price paid/received per share from trading x shares ($x > 0$ is a buy and $x < 0$ is a sell). Then, the market equilibrium condition is modified to

$$E_0 + mx = \begin{cases} [E_0 + cN] - \frac{Nq}{1+r} + \beta & if \quad sufficient\ capital \\ [E_0 + cN](1-q) + \beta & if \quad insufficient\ capital \end{cases}. \tag{18}$$

Solving for the equilibrium insurance premium

$$c = \begin{cases} \frac{\phi p}{(1+r)} + \frac{mx-\beta}{N} & if \quad sufficient\ capital \\ \frac{E_0 \phi p + mx - \beta}{N(1-\phi p)} & if \quad insufficient\ capital \end{cases}. \tag{19}$$

And finally the implied mortality probability is

$$p_{implied} = \begin{cases} \left(c - \frac{mx-\beta}{N}\right)\frac{(1+r)}{\phi} & if \quad sufficient\ capital \\ \frac{cN - mx + \beta}{(E_0 + cN)\phi} & if \quad insufficient\ capital \end{cases}. \tag{20}$$

As shown, the introduction of these market realities makes the estimation of the implied mortality probability even more complex and dependent upon additional market parameters related to market frictions.

Last, in very illiquid markets where the competitive market assumption doesn't hold, strategic trading considerations and market manipulation are possible. In this case the equilibrium market price depends on the state contingent trading strategies of the market participants, which are usually unobservable (see Jarrow [8]). Consequently, no simple formulas such as those in expression (20) are available, and it is not possible to accurately determine an implied mortality probability using market prices.

The complexity of the estimation under realistic market frictions and short sale constraints, as documented in expression (20), leads to a few observations about implied mortality probabilities:

1. they crucially depend on the model used, and
2. they are a complex non-linear function of the insurance premium depending on market parameters related to risk premium, counterparty risk, short sale restrictions, and transaction costs.

Extending the simple single period model to a dynamic economy with a stochastic term structure of interest rates will change the exact formulas obtained, but the two qualitative conclusions reached above remain intact (see Jarrow [11]).

Remark 8 *Short sale restrictions are equivalent to banning naked CDS, i.e. buying a CDS without holding the underlying debt issue.*

6 Why Use Implied Mortality Probabilities?

Given the above insights, we can now answer this question. There are only two reasons why one might use implied mortality probabilities instead of actuarial based probability estimates: (1) ease of computation, and (2) market efficiency considerations. We explain each of these reasons in turn, showing that neither provides a valid justification for using implied mortality probabilities.

6.1 Ease of Computation

Market insurance premiums are readily available, and given the *simplest* model, it is an easy exercise to compute implied mortality probabilities. The simplest model is

$$p_{implied} = c(1 + r) \tag{21}$$

where risk premium, counterparty risk, market frictions, and strategic trading are ignored. In contrast, it is much more difficult to collect actuarial data on population mortalities, to condition on an insured's characteristics, and compute a mortality probability estimate. This logic is true for the simplest model, but unfortunately the simplest model is not a valid representation of reality. If used, the resulting probability estimates are misspecified and significantly biased (as shown above).

Instead, to obtain an unbiased estimate, one needs to use a *valid* model which is consistent with the existing market structure. In this case, the implied probability estimate is very difficult to obtain. In addition to the market insurance premiums as an input, one needs to also estimate parameters relating to risk premium, counterparty risk, and the market frictions. These parameters are difficult to estimate correctly, and with respect to the risk premium, there is no consensus on how to do this even after serious academic study for over 50 years.

Remark 9 *The simplest model given in expression (21) is commonly applied in the CDS market to estimate implied default probabilities without a risk premium adjustment.*

6.2 Market Efficiency Considerations

If equity markets are efficient, prices reflect information not widely known, due to the trading activity of informed traders (see Fama [7] and Jarrow and Larsson [12]). And if one can invert market prices to obtain the implied mortality probabilities, perhaps these estimates will include more information than that which is available using actuarial data alone? Although true conceptually, this argument fails for two reasons.

First, to get the implied mortality probabilities using market prices, one must first identify a valid model, which is consistent with the existing market structure. And given such a model, then one needs to estimate the remaining market parameters. We have already discussed that this is nearly an impossible task.

Second, there is another hurdle. How does one determine if a model is valid? To determine the validity of the model, one must use actuarial data in conjunction with market prices to test the model. Only if the model is accepted by actuarial data should it be used for estimating implied probabilities. But if a correct model is estimated and fit using actuarial mortality data, then the implied and actuarial probability estimates will be identical, making irrelevant the need to determine implied probabilities (see Jarrow [10] for an elaboration of this argument).

7 Conclusion

When looking at mortality probability estimation, common practice is to use actuarial mortality tables based on historical population mortalities. It seems absurd to instead use life insurance premiums to infer the mortality probabilities embedded within them. The logic underlying this common practice is correct and the reasons for its validity are documented in this paper. To use implied mortality probabilities, one must model life insurance premiums including risk premium, counterparty risk, market frictions, and strategic trading. Practically speaking, this is a very difficult if not an impossible task.

Surprisingly, when discussing corporate or sovereign default probabilities, the common belief is almost the reverse. For some unknown reason, it is believed that implied default probabilities from CDS spreads provide reliable estimates. This paper shows that this common belief regarding implied default probabilities is false. This is accomplished by showing an exact analogy between life insurance premiums and CDS spreads, so that the life insurance based intuition can be applied.

Due to the Dodd-Frank Wall Street Reform and Consumer Protection Act credit ratings are no longer going to be included in U.S. financial regulations. Hence, U.S. regulators are searching for an alternative method for accessing default probabilities and credit risk. CDS implied default probabilities are being considered. But as shown above, this is a foolish alternative. The most obvious choice is the analogue to actuarial based mortality probabilities, actuarial based default probabilities conditioned on the characteristics of the firm under consideration (see Chava and Jarrow [4], Campbell, Hilscher, Szilagyi [2], Bharath and Shumway [1]).

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