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Australian School of Business Research Paper No. 2014ACTL01

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Last updated: 11/02/14 CRICOS Code: 00098G

Reverse Mortgage Pricing and Risk Analysis Allowing for Idiosyncratic House Price Risk and Longevity Risk

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February 7, 2014

Abstract

Reverse mortgages provide an alternative source of funding for retirement income and health care costs. The two main risks that reverse mortgage providers face are house price risk and longevity risk. Recent real estate literature has shown that the idiosyncratic component of house price risk is large. We analyse the combined impact of house price risk and longevity risk on the pricing and risk profile of reverse mortgage loans in a stochastic multi-period model. The model incorporates a new hybrid hedonic-repeat-sales pricing model for houses with specific characteristics, as well as a stochastic mortality model for mortality improvements along the cohort direction (the Wills-Sherris model). Our results show that pricing based on an aggregate house price index does not accurately assess the risks underwritten by reverse mortgage lenders, and that failing to take into account cohort trends in mortality improvements substantially underestimates the longevity risk involved in reverse mortgage loans.

Keywords: Equity release products; idiosyncratic house price risk; stochastic mortality; Wills-Sherris mortality model

JEL Classifications: G21, G22, G32, R31, L85

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1 Introduction

A growing literature addresses the pricing and risk management of reverse mortgages and other equity release products. More and more sophisticated pricing techniques are being used and a range of different models have been developed for the health-related termination of equity release products. Several studies including Wang *et al.* (2008), Li *et al.* (2010) and Yang (2011) assess the impact of longevity risk on the pricing and risk management of reverse mortgages.

A key risk factor - house price risk - has received relatively less research attention. Previous studies have typically assessed house price risk based on market-wide house price indices. For example, Chen *et al.* (2010), Yang (2011) and Lee *et al.* (2012) model house price risk using a nationwide house price index for the United States, whereas Hosty *et al.* (2008) and Li *et al.* (2010) use a nationwide index for the UK. Wang *et al.* (2008) average house prices in eight capital cities in Australia. Sherris and Sun (2010), Alai *et al.* (2013) and Cho *et al.* (2013) use city-level data for Sydney, Australia. Reverse mortgage loans implicitly include no-negative equity guarantees that are basically a portfolio of options on individual properties, instead of an option on a portfolio of properties. Therefore, pricing reverse mortgage loans based on aggregate house price data does not take into account the large idiosyncratic component in house price risk. Recent real estate research has shown that the trends and risks in houses prices vary substantially across different submarkets within a city (see, e.g., Bourassa *et al.*, 1999, 2003; Ferreira and Gyourko, 2012; Hanewald and Sherris, 2013). Standard property valuation techniques take into account the characteristics of the property and of the surrounding neighbourhood (see, e.g., Shao *et al.*, 2013, for a recent literature review). One major reason that idiosyncratic house price risk is not widely accounted for in the current literature is the limited public access to individual house transactions data (Li *et al.*, 2010).

The aim of our study is to assess how idiosyncratic house price risk and longevity risk impact the pricing and risk analysis of reverse mortgage loans. We model house price risk using a hybrid hedonic-repeat-sales model for projecting future values of properties with specific characteristics (Shao *et al.*, 2013). The model is estimated using a large data set on individual property transactions. Different mortality assumptions are tested to assess the impact of longevity risk: we compare the results obtained using deterministic mortality improvements and two different stochastic mortality models respectively developed by Cairns *et al.* (2006) and Wills and Sherris (2008). We also test the sensitivity of the results with respect to the assumptions on non-mortality related causes of reverse mortgage termination, including entry into long-term care, prepayment and refinancing. We use the pricing technique developed in a recent paper by Alai *et al.* (2013). Our paper extends the work presented in the six-page conference paper by Shao *et al.* (2012), where the same house price model was used to study the impact of idiosyncratic house price risk on reverse mortgage pricing, but mortality rates employed in Shao *et al.* (2012) were taken from the 2008 period life table of the Australian population without taking into account mortality improvements.

The results of our study show that pricing reverse mortgage loans based on an average house price index results in a substantial misestimation of the risks in reverse mortgages. The financial risks are underestimated for reverse mortgage loans issued with low loan-to-value ratios and overestimated for loans with high loan-to-value ratios. Longevity risk is another important risk factor. The comparison of the different mortality models shows that the key factor is the assumption with respect to the trend, rather than with respect to the uncertainty of future mortality rates. The results are found to be relatively robust to the assumptions on non-mortality causes of termination.

The remaining part of this paper is arranged as follows. Section 2 describes the data. Section 3 develops a pricing framework for reverse mortgages allowing for idiosyncratic house price risk and longevity risk. We explain how to estimate and project disaggregated house price indices and stochastic discount factors and describe the stochastic model used to forecast future mortality rates. Based on these building blocks, values of No-Negative Equity Guarantees (NNEG) embedded in reverse mortgage loans and the mortgage insurance premium rates are calculated in Section 4. Robustness tests are also performed to test whether the results are sensitive to the assumptions with respect to termination rates. The last section concludes.

2 Data

Our study is based on Australian data. Residex Pty Ltd, a Sydney-based company, provides a large data set containing individual house transactions in the Sydney Statistical Division over the period 1971-2011. Sydney is the largest city in Australia. About one fifth of Australia's population resided in the Sydney Statistical Division as of June 2010 according to numbers published by the Australian Bureau of Statistics. We also use data on Sydney rental yield rates obtained from Residex, Australian GDP growth rates from the Australian Bureau of Statistics, and zero-coupon bond yield rates from the Reserve Bank of Australia. The economic time series are available for the period 1992-2011.

We use mortality rates for the Australian male and female population aged 50-109, obtained from the Human Mortality Database. Mortality data are available for the period 1921-2009, but only data for 1970-2009 are used due to the obvious change in mortality trends before and after 2007. Cocco and Gomes (2012) document that the average annual increases in life expectancy are much larger after 1970 than before in eight OECD countries. To investigate whether a similar trend break occurs in Australian data, we plot the averaged log mortality rates (averaged across age groups) for males and females in Fig. 1. There is a noticeable trend break in the early 1970s. We therefore use mortality data from 1970 onward. This choice is also justified by changes in the reporting of Australian population and death statistics in 1971 (Andreeva, 2012).

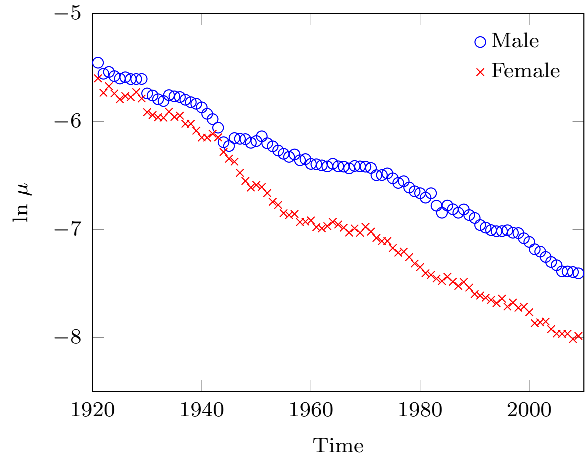


Fig. 1. Average log mortality rates for Australian males and females, ages 50-100, 1921-2009.

3 A reverse mortgage pricing framework allowing for idiosyncratic house price risk and longevity risk

3.1 The reverse mortgage contract

We model a reverse mortgage loan with variable interest rates and a single payment at issuance. The outstanding loan amount accumulates until the borrower dies or permanently leaves the house due to non-mortality reasons. This contract design is the most common form of equity release products in the United States (Consumer Financial Protection Bureau, 2012) and in Australia (Deloitte and SEQUAL, 2012). We focus on single female borrowers who are the most common reverse mortgage borrowers in the United States (Consumer Financial Protection Bureau, 2012). In Australia, the majority of reverse mortgage borrowers are couples, and single females are the second most common borrowers (Deloitte and SEQUAL, 2012).

The outstanding reverse mortgage loan amount for a single borrower aged x is a function of the random termination time T_x:

$$L_{T_x} = L_0 \exp\left(\sum_{t=1}^{K_x+1} \left(r_t^{(1)} + \kappa + \pi \right) \right), \tag{1}$$

where L_0 is the issued net loan amount, $K_x = [T_x]$ is the curtate termination time of the contract, $r_t^{(1)}$ is the quarterly risk-free rate which is assumed to be the one-quarter zero-coupon bond yield rate, κ is the quarterly lending margin, and π is the quarterly mortgage insurance premium rate charged following the assumptions in Chen *et al.* (2010). The loan-to-value ratio is defined as the ratio of the loan amount L_0 to the house price H_0 at the issuance of the loan.

We assume that the property can be sold immediately when the contract is terminated. The sale proceeds (less transaction costs) are used to repay the outstanding loan and the remaining amount goes to the borrower's estate. In case the sale proceeds are insufficient to repay the outstanding loan, the lender or the lender's insurer are responsible for the shortfall. The risk that the loan balance exceeds the house price at termination is referred to as cross-over risk.

Reverse mortgages in the United States and in Australia include a No-Negative Equity Guarantee (NNEG), which caps the borrower's repayment at the house price H_{T_x} at the time of termination T_x. The net home equity of the borrower is the property value less the required loan repayment:

$$\text{Net Equity}_{T_x} = H_{T_x} - \min\{L_{T_x},\ H_{T_x}\} = \max\{H_{T_x} - L_{T_x},\ 0\}, \tag{2}$$

which guarantees that the net home equity of the borrower is non-negative and gives an explicit description of the no-negative equity guarantee (NNEG) in the reverse mortgage loan. The NNEG protects the borrower against the downside risk in future house prices. The guarantee is comparable to a put option with the collateralised property as the underlying asset and an increasing strike price (see, e.g., Chinloy and Megbolugbe, 1994).

This paper adopts the valuation approach developed in Alai *et al.* (2013) and applied in Cho *et al.* (2013) to value reverse mortgage loans from the lender's perspective. We calculate the values of NNEG based on quarterly payments. The possible loss of the lender is a function with respect to future house prices:

$$\text{Loss}_{T_x} = \max\{L_{T_x} - (1-c)H_{T_x},\ 0\} \prod_{s=1}^{K_x+1} m_s, \tag{3}$$

where c captures the transaction cost in the sale of properties, and m_s is the risk-adjusted discount factor during the s^{th} quarter.

The NNEG is the expected present value of the provider's expected future losses:

$$\text{NNEG} = \sum_{t=0}^{\omega-x-1} E\left[{}_{t|}q_x^c \ \text{Loss}_{t+1}\right], \tag{4}$$

where ω is the highest attainable age, and ${}_{t|}q_x^c = \Pr(t < T_x \leq t+1)$ is the probability that the contract is terminated between t and $t+1$. Extending the work by Alai *et al.* (2013) and Cho *et al.* (2013), we allow the termination probability to be a random variable. Following these papers, we assume that the costs for providing the NNEG are charged to the borrower in the form of a mortgage insurance premium with a fixed premium rate π accumulated on the outstanding loan amount. The expected present value of the accumulated mortgage insurance premium is given by:

$$\text{MIP} = \pi \sum_{t=0}^{w-x-1} E\left[{}_tp_x^c \ L_t \prod_{s=0}^{t} m_s\right], \tag{5}$$

where ${}_tp_x^c = \Pr(T_x > t)$ is the probability that the contract is still in effect at time t, and m_0 is defined to be 1. The accumulated mortgage insurance premium should be sufficient to fund the losses arising to the lender from the embedded NNEG. We calculate the value of quarterly mortgage insurance premium rate π by equating NNEG and MIP.

To assess the impact of stochastic mortality, we also calculate the difference between the NNEG and MIP, but allowing for uncertainty in the probabilities ${}_{t|}q_x^c$ and ${}_{t}p_x^c$. We denote this shortfall as SF:

$$\text{SF} = \sum_{t=0}^{\omega-x-1} \left[{}_{t|}q_x^c \ \text{Loss}_{t+1} \right] - \pi \sum_{t=0}^{w-x-1} \left[{}_{t}p_x^c \ L_t \prod_{s=0}^{t} m_s \right]. \tag{6}$$

Note that the two terms in Equation (6) differ from NNEG and MIP by the missing expectation operator after the summation symbols. SF is expected to have an expected value of zero but the dispersion can be large if mortality shows very volatile improvements. The Tail Value-at-Risk of the shortfall are used to assess the impact of stochastic mortality on the risks underwritten by reverse mortgage providers.

3.2 The hybrid house price model

In the residential house price literature, the value of a house, V_{it}, is generally expressed as $V_{it} = Q_{it}P_t$, where Q_{it} is the quality measure of the house and P_t is the house price index (Englund *et al.*, 1998; Quigley, 1995). A range of models are developed to disentangle the two components. Standard models include the hedonic model, the repeat-sales model and the hybrid hedonic-repeat-sales model. A typical hedonic model expresses the logarithm of the house price as a function of a property's characteristics, locations, amenities, and other variables that add values to the house (Bourassa *et al.*, 2011). The hedonic model has the heterogeneity problem and possible specification errors. The repeat-sales model addresses these shortcomings by differencing the regression equation in the hedonic model. The repeat-sales model requires observations on properties that are transacted multiple times. The hybrid hedonic-repeat-sales house price model, first proposed by Case and Quigley (1991), combines the advantages of the hedonic model and the repeat-sales model.

A recent study by Shao *et al.* (2013) compares methods of constructing disaggregated house price indices and develops a new hybrid hedonic-repeat-sales house price model. The model is given by three stacked equations. The first equation is a modified hedonic house price regression on houses that are transacted only once in the sample period. The regression can be expressed as follows:

$$V_{it} = \alpha + T'\beta + X'\gamma + X'\Delta T + \eta_i + \xi_{it}, \tag{7}$$

where V_{it} is the natural logarithm of the value of an individual house i at time t, α is the intercept, T is a vector of time dummy variables, X is a vector of property characteristics, β and γ are vectors of coefficients, Δ is a matrix of coefficients of the interactions between time dummy variables and house characteristics, η_i captures the specification error, and ξ_{it} is the disturbance term. The sum of the specification error and the disturbance term is denoted by $\varepsilon_{it} = \eta_i + \xi_{it}$.

The second stacked equation is to use Equation (7) again on houses that are transacted more than once, but excluding the last sale of each property.

The third stacked equation is the differenced Equation (7), which expresses the differenced log house prices with respect to time dummy variables and their interaction terms with house characteristics:

$$V_{it} - V_{is} = D'\beta + X'\Delta D + \xi_{it} - \xi_{is}, \tag{8}$$

where D is a vector containing the differenced time dummy variables. If the first sale is at time s and the second at t, the values of the s^{th} and t^{th} components in D are respectively -1 and 1 with other components being zeros.

For houses i and j ($i \neq j$), the assumptions with respect to the error terms are:

$$
\begin{aligned}
&E(\xi_{it}) = 0, E(\eta_i) = 0 \ ; \\
&E(\xi_{it}^2) = \sigma_\xi^2, E(\eta_i^2) = \sigma_\eta^2 \ ; \\
&E(\xi_{it}\xi_{js}) = 0 \text{ if } (i-j)^2 + (t-s)^2 \neq 0 \ ; \\
&E(\eta_i\eta_j) = 0 \text{ if } i \neq j; \\
&E(\eta_i\xi_{it}) = 0.
\end{aligned} \tag{9}
$$

Based on the above assumptions, the covariance matrix of the three stacked equations that accounts for the dependence between repeated sales of the same property can be expressed as follows:

$$
Cov = \begin{pmatrix} \sigma_\varepsilon^2 I_M & 0 & 0 \\ 0 & \sigma_\varepsilon^2 I_N & -\sigma_\xi^2 I_N \\ 0 & -\sigma_\xi^2 I_N & 2\sigma_\xi^2 I_N \end{pmatrix}, \tag{10}
$$

where σ_ε^2 is the variance of ε_{it}, σ_ξ^2 is the variance of ξ_{it}, M is the number of houses with single transactions, N is the number of pairs of repeat-sales in the repeat-sales equation, and I_d denotes a d-dimensional identity matrix. It can be shown that the variance of ε_{it} is $\sigma_\varepsilon^2 = \sigma_\eta^2 + \sigma_\xi^2$.

We estimate a version of the hybrid hedonic-repeat-sales model developed by Shao *et al.* (2013) that includes more detailed geographic variables in the vector X containing the property's characteristics. In particular, we additionally include a property's geographic coordinates (longitude, latitude) and dummy variables indicating whether the property's postcode area is located directly next to the central business district, the Sydney harbour, the coastline, a park or an airport. To avoid multicollinearity problems, we exclude the postcode dummy variables that are included in the model estimated by Shao *et al.* (2013). Another difference is that we use the hybrid model to estimate monthly house price indices, while Shao *et al.* (2013) focus on yearly indices.

The estimated parameters γ_t and β_t are used to construct the aggregate house price index using the following equation:

$$P_t = 100 \exp\left(\gamma_t + \bar{X}_0 \beta_t\right), \tag{11}$$

where $\bar{X}_0$ is a row vector of average values of characteristics in the base year. The price index for a particular type of house, denoted as k, is calculated as:

$$P_t^k = 100 \exp\left(\gamma_t + X^k \beta_t\right) = P_t \exp\left(\left(X^k - \bar{X}_0\right) \beta_t\right), \tag{12}$$

where X^k is a row vector of the characteristic variables for houses of the type k.

3.3 Projection of future house prices and discount factors

This section projects future house price indices based on the historical indices constructed in Section 3.2. Stochastic discount factors are then generated based on the projections.

3.3.1 Aggregate house price index projection

Following Alai *et al.* (2013) and Cho *et al.* (2013), a Vector Auto-Regression (VAR) model is used to project future average house price growth rates and future risk-adjusted discount factors. The VAR model is given by:

$$Y_t = \kappa + \sum_{i}^{p} \Phi_i Y_{t-i} + \Sigma^{1/2} Z_t, \tag{13}$$

where Y_t is a vector of K state variables, p is the lag length in the model, Φ_i is a K-dimensional matrix of parameters, Σ is the covariance matrix, $\Sigma^{1/2}$ is the Cholesky decomposition of Σ, and Z_t is a vector of independently distributed standard normal variables.

Five state variables are included in the model ($K = 5$): one-quarter zero-coupon bond yield

Table 1. Information criteria for VAR models with different lags.

Criterion	VAR(1)	VAR(2)	VAR(3)	VAR(4)
AIC	-15.862	-17.547	**-17.552**	-17.394
BIC	-14.935	**-15.834**	-15.042	-14.073
AICc	-15.792	**-17.287**	-16.937	-16.193

rates $r_t^{(1)}$, the spread of five-year[1] over one-quarter zero-coupon bond yield rates $r_t^{(20)} - r_t^{(1)}$, Australian GDP growth rates g_t, Sydney average house price index growth rates h_t, and Sydney rental yield rates y_t. All the variables are converted to continuously compounded quarterly rates. The data covers the period Sep-1992 to Jun-2011. Individual house price indices are not included in this VAR model since individual risk should not be priced according to the CAPM theory. Systematic mortality risk is also not priced in the model, following the assumption of independence between mortality and macroeconomic variables as, for example, in Blackburn and Sherris (2013).[2]

The optimal lag length of the VAR model is selected based on three commonly used information criteria: Akaike's information criterion (AIC), the Schwarz-Bayesian information criterion (BIC) and Akaike's information criterion corrected for small sample sizes (AICc). BIC puts more values on the parsimony of the model setup than AIC. AICc addresses the problem of the small sample size compared to the larger number of parameters involved in the VAR model. The values of the three information criteria are shown in Table 1. Although AIC suggests an optimal lag length of three, the value for VAR(2) is very close, and both BIC and AICc show that the optimal lag length is two. The estimation results for the parameters Φ_1 and Φ_2 and the covariance matrix Σ in the VAR(2) model are given in Table 2.

[1]Alai *et al.* (2013) and Cho *et al.* (2013) use the spread of ten-year over one-quarter zero-coupon bond yield rates. We compared VAR models based on Akaike's information criterion and Schwarz's Bayesian information criterion, and choose the spread of five-year over one-quarter zero-coupon bond yield rates.

[2]Several studies report significant short-term correlations between mortality rates and macroeconomic indicators such as GDP growth rates and unemployment rates. The pro-cyclical link has been explained by a causal effect of economic conditions on mortality rates (see, e.g., Granados *et al.*, 2008; Hanewald, 2011; Ruhm, 2007). We are not aware of a study documenting a causal effect of mortality rates on asset prices and risk premia in the real estate or reverse mortgage market.

Table 2. Parameter estimates and covariance matrix in the VAR(2) model.

	Parameter Estimates				
Variable	$r_t^{(1)}$	$r_t^{(20)} - r_t^{(1)}$	g_t	h_t	y_t
Constant	0.165*	0.101	1.267***	1.442	-0.018
$r_{t-1}^{(1)}$	1.272***	-0.369***	0.852***	-0.603	-0.010
$r_{t-1}^{(20)} - r_{t-1}^{(1)}$	0.304**	0.781***	-0.240	3.667	-0.069
g_{t-1}	0.013	-0.002	1.141***	-0.223	0.005
h_{t-1}	0.010	-0.012	0.008	-0.134	-0.003
y_{t-1}	0.850**	-0.092	0.725	-6.758	1.196***
$r_{t-2}^{(1)}$	-0.306**	0.177	-0.685**	-0.672	0.027
$r_{t-2}^{(20)} - r_{t-2}^{(1)}$	-0.047	-0.125	-0.014	-6.614***	0.093*
g_{t-2}	-0.020	-0.001	-0.839***	-0.658	0.008
h_{t-2}	0.014*	-0.001	0.003	0.510***	-0.003
y_{t-2}	-0.996***	0.276	-0.997	9.228	-0.216
	Covariance Matrix				
Variable	$r_t^{(1)}$	$r_t^{(20)} - r_t^{(1)}$	g_t	h_t	y_t
$r_t^{(1)}$	0.011^{+}	-0.002	0.013^{+}	$-0.016^{\cdot}$	0.000
$r_t^{(20)} - r_t^{(1)}$	-0.002	0.011^{+}	-0.003	0.007	0.001^{+}
g_t	0.013^{+}	-0.003	0.045^{+}	0.026	-0.001
h_t	-0.016	0.007	0.026	3.250^{+}	-0.017^{-}
y_t	0.000	0.001^{+}	-0.001	-0.017^{-}	0.001^{+}

* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$. $^{+}$ > 2*std error; $^{-}$ < 2*std error.

3.3.2 Stochastic discount factors

Following Alai *et al.* (2013), stochastic discount factors that reflect the key risks in reverse mortgage cash flows are used to value the reverse mortgage. The discount factor is modelled as:

$$m_{t+1} = \exp(-r_t^{(1)} - \lambda_t'\lambda_t - \lambda_t' Z_{t+1}), \tag{14}$$

where λ_t is the time-varying market price of risk, which is assumed to be an affine function of the state variables Y_t in the VAR (2) model (Ang and Piazzesi, 2003):

$$\lambda_t = \lambda_0 + \lambda_1 Y_t. \tag{15}$$

To derive stochastic discount factors based on the VAR model, zero-coupon bond prices are assumed to be exponential linear functions of contemporaneous and one-quarter lagged state variables (Shao *et al.*, 2012):

$$p_t^n = \exp(A_n + B_n' Y_t + C_n' Y_{t-1}), \tag{16}$$

where A_n, B_n and C_n are parameters that can be solved for using the following differenced equations (proof in Shao *et al.*, 2012):

$$\begin{cases} A_{n+1} = A_n + B_n'(\kappa - \Sigma^{1/2}\lambda_0) + \dfrac{1}{2} B_n' \Sigma B_n, \\ B_{n+1} = (\Phi_1 - \Sigma^{1/2}\lambda_1)' B_n + C_n - e_1, \\ C_{n+1} = \Phi_2' B_n. \end{cases} \tag{17}$$

The initial values of the three parameters in Equations (17) are $A_1 = 0$, $B_1 = -e_1$, and $C_1 = 0$, where e_1 denotes a vector with the first component of one and other components of zeros. The estimated quarterly yield rate with n quarters to maturity at time t is expressed

Table 3. Correlation coefficients between stochastic discount factors and state variables.

Variable	m_t	$r_t^{(1)}$	$r_t^{(20)} - r_t^{(1)}$	g_t	h_t	y_t
m_t	1.000					
$r^{(1)}$	-0.940***	1.000				
$r_t^{(20)} - r_t^{(1)}$	0.235**	-0.261**	1.000			
g_t	-0.296**	0.396***	-0.362***	1.000		
h_t	0.108	-0.298***	0.346***	-0.155	1.000	
y_t	-0.315***	0.262**	0.602***	-0.299***	0.202*	1.000

* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$.

as $\hat{r}_t^{(n)} = -(A_n + B_n' Y_t + C_n' Y_{t-1})/n$. The market price of risk is obtained by minimising the sum of squared deviations of the estimated yield rates from the observed rates:

$$\min_{\lambda} \left\{ \sum_n \left(\hat{r}_t^{(n)} - r_t^{(n)} \right)^2 \right\}. \tag{18}$$

Equations (14) and (15) link the stochastic discount factors to the state variables in the VAR model. Table 3 reports the correlations between the estimated stochastic discount factors and the state variables. All correlations, except those with the growth rates of the aggregate house price index, are economically and statistically significant. The estimated stochastic discount factors reflect the risks in the state variables.

3.3.3 Disaggregated house price index projection

Disaggregated house price indices for properties with specific characteristics are then linked to the average house price index using a VAR with exogenous variables (VARX($\tilde{p}, \tilde{q}$)) model, where the average house price index is the exogenous variable. The model is given by:

$$h_t^d = \tilde{\kappa} + \sum_{i=1}^{\tilde{p}} \tilde{\Phi}_i \; h_{t-i}^d + \sum_{j=0}^{\tilde{q}} \tilde{\Omega}_j \; h_{t-j} + \tilde{\Sigma}^{1/2} \tilde{Z}_t, \tag{19}$$

where h_t^d is a vector of growth rates of the disaggregated house price indices, h_t is the growth rate of the aggregate house price index, $\tilde{Z}_t$ is a vector of independent standard normal random variables, $\tilde{p}$ is the lag length for the state variables, and $\tilde{q}$ is the lag length for the exogenous variable. The optimal lag lengths for the VARX model are selected based on the three information criteria reported in Table 4. All three criteria suggest that a VARX(1,0) is the optimal specification.

Table 4. Information criteria for VARX models with different lag lengths $(\tilde{p}, \tilde{q})$.

Criteria	(1,0)	(1,1)	(1,2)	(2,0)	(2,1)	(2,2)
AICC	**9.927**	10.231	10.533	11.643	12.131	12.366
AIC	**9.317**	9.504	9.650	9.327	9.428	9.552
BIC	**13.025**	13.521	14.009	15.977	16.489	16.725

Using the estimates of the VARX(1,0) model, price indices for houses with specific characteristics are simulated. The simulation accounts for parameter uncertainty in the VARX model. We assume that the two parameters $\tilde{\Phi}_i$ and $\tilde{\Omega}_j$ are independent and are normally distributed and estimate the variance based on the sample. Fig. 2 shows the projected aggregate price index for Sydney and the indices for houses located in different regions of Sydney, including the central business district, the coastline, the Sydney harbour, the airport vicinity, and suburbs that have a park. The regional classification is based on the postcode area classification in Hanewald and Sherris (2013). The prices of houses in these different regions are more volatile than the average house prices in Sydney.

3.4 Termination of reverse mortgages

Termination triggers of reverse mortgages include mortality, move-out due to health-related issues, voluntary prepayment and refinancing (Ji *et al.*, 2012). To model the different termination triggers we use a variant of the multi-state Markov model developed by Ji *et al.* (2012). Similar models have been used by Alai *et al.* (2013) and Cho *et al.* (2013). We

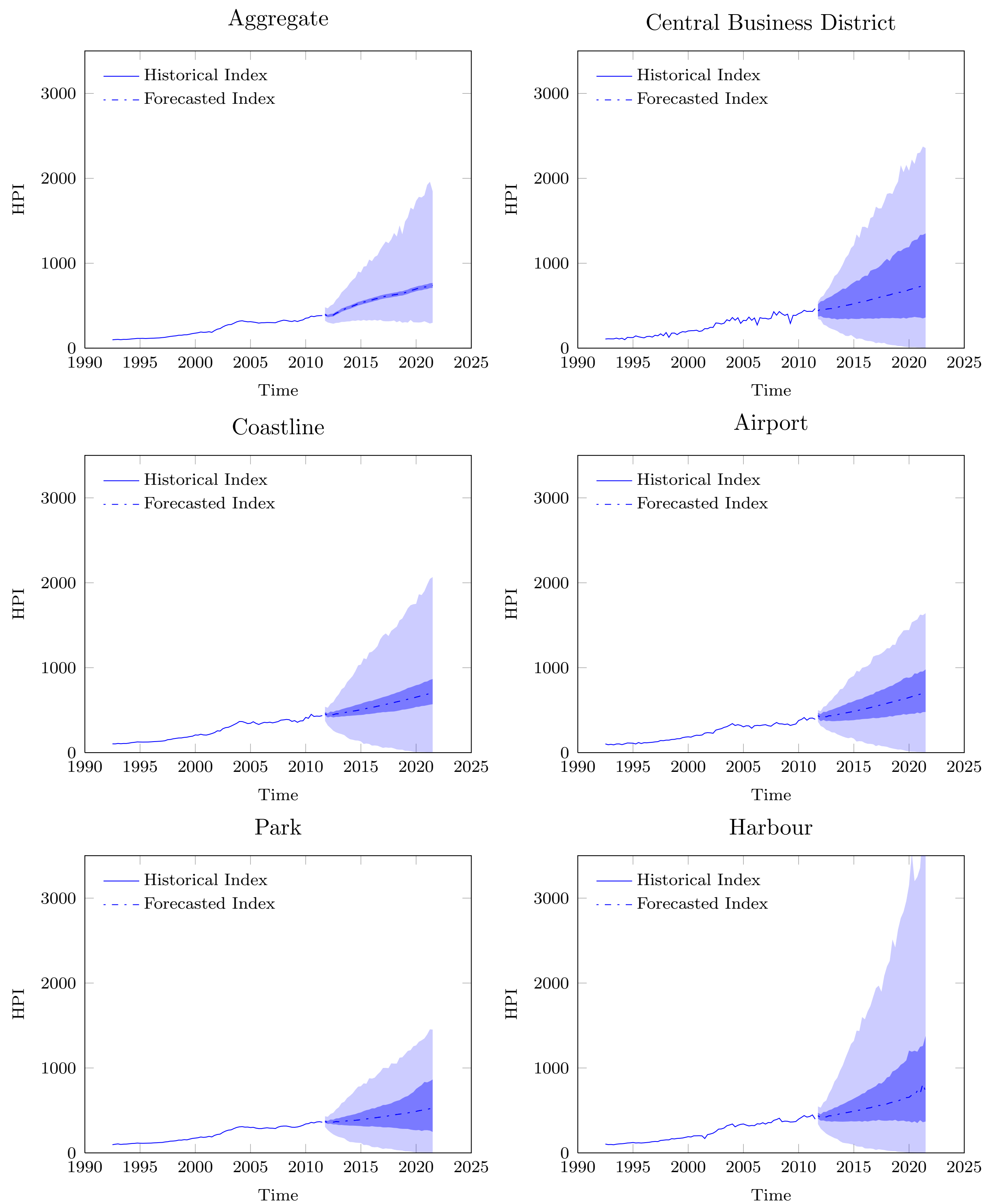


Fig. 2. Projection of price indices for houses in different regions of Sydney. Dark-shaded areas represent 95%-confidence intervals without incorporating parameter uncertainty. Light-shaded areas are 95%-confidence intervals that take into account parameter uncertainty.

extend this line of research by allowing mortality rates to be random variables. In the following, we first describe the calibration of the Wills-Sherris stochastic mortality model and the projection of future mortality rates. We then describe the modelling of the termination triggers.

3.4.1 Stochastic mortality

Wills and Sherris (2008) develop a multi-variate stochastic mortality model to describe the volatile improvement of mortality rates over time. The model describes changes in age-specific mortality rates along the cohort direction as a function of age, time effects and multiple stochastic risk factors. Observed correlations between the year-to-year changes in mortality rates of different age groups are incorporated in the multivariate distribution of the stochastic risk factors. The Wills-Sherris model allows for a more flexible and realistic age dependence structure than, for example, the one-factor model by Lee and Carter (1992) and the two-factor model by Cairns *et al.* (2006). An explicit expression for the age dependence structure can be derived in the Wills-Sherris model. The Wills-Sherris model has been applied in several studies analysing the pricing and risk-management of financial products exposed to longevity risk (see, e.g. Hanewald *et al.*, 2012; Ngai and Sherris, 2011; Wills and Sherris, 2010).

The model is formulated for the changes of log mortality rates along the cohort direction: $\Delta_c \ln \mu(x,t) = \ln \mu(x,t) - \ln \mu(x-1,t-1)$ where $\mu(x,t)$ is the force of mortality for a person aged x at time t with $x = x_1, x_2, \cdots, x_N$ and $t = t_1, t_2, \cdots, t_T$. These cohort changes are assumed to follow:

$$\Delta_c \ln \mu(x,t) = ax + b + \sigma\varepsilon(x,t), \tag{20}$$

where a, b and σ are parameters to be estimated, and $\varepsilon(x,t)$ follows a standard normal distribution that drives the fluctuation of mortality improvements. To account for age dependence, $\varepsilon(x,t)$ is expressed as a linear combination of independent standard normal random variables: $\boldsymbol{\varepsilon}_t = [\varepsilon(x_1,t), \varepsilon(x_2,t), \cdots, \varepsilon(x_N,t)]' = \Omega^{\frac{1}{2}} W_t$, where Ω is a covariance matrix that reflects the age dependence structure and W_t is a vector of independent standard normal random variables.

In the Wills-Sherris model, the log changes in mortality rates along the cohort direction are mixed effects that can be decomposed into age and period effects. Fig. 3 shows the average values (averaged over time) of the log changes in mortality rates along the cohort direction for different ages. There is a linear trend in these log changes, providing justification for the specification of the Wills-Sherris model. Similar to the Lee-Carter model and the two-factor CBD model, mortality changes can be decomposed into age and time effects. The decomposition of the mixed effects in the Wills-Sherris model is shown in the following equation:

$$\Delta_c \ln \mu(x,t) = [ax + b - g(x)] + [g(x) + \sigma\varepsilon(x,t)], \tag{21}$$

where $g(x)$ is an implicit function of age that captures the trend of the stochastic improvements in mortality rates over time. For example, if $g(x)$ is lower for smaller x, it suggests that mortality improvements are more pronounced for younger ages. In Equation (21), $[ax + b - g(x)]$ is the age effect and $[g(x) + \sigma\varepsilon(x,t)]$ captures the period effect. For mortality projections, we are not interested in the value of $g(x)$ since the effect of $g(x)$ is cancelled in the cohort direction.

The Wills-Sherris model is estimated for male and female mortality rates for ages 50-100 and years 1970-2009 using a linear regression of Equation (20). The estimated parameters $\hat{a}$, $\hat{b}$ and $\hat{\sigma}$ are given in Table 5, and the covariance matrix of the parameters is shown in Table 6. The estimate for a is negative for males and positive for females. This is consistent with Fig. 3, where the log changes in mortality rates show a slightly downward sloping trend for males and an positive trend for females. This pattern implies that over the sample period the changes in log mortality rates along the cohort direction have been larger for males at younger ages and for females at older ages.

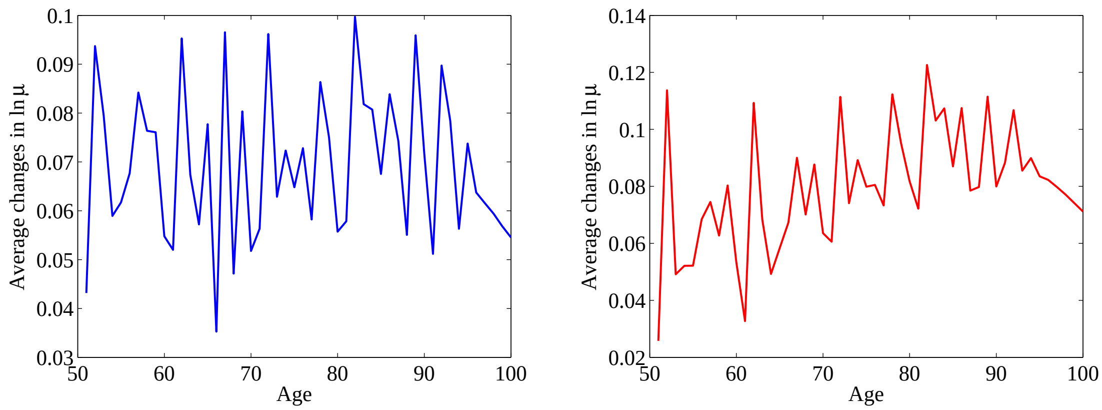


Fig. 3. Average changes in log mortality rates along the cohort direction for different ages for Australian males (left) and females (right), 1970-2009.

Table 5. Parameter estimates for the Wills-Sherris model based on data for Australia, 1970-2009.

Parameter	Male	Female
$\hat{a}$ ($\times 10^{-4}$)	-16.94	6.39***
$\hat{b}$ ($\times 10^{-2}$)	7.07***	3.12**
$\hat{\sigma}$ ($\times 10^{-2}$)	6.15***	6.73***

* $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

Table 6. Covariance matrix of estimated parameters in the Wills-Sherris model.

Parameter	Male		Female	
	$\hat{a}$	$\hat{b}$	$\hat{a}$	$\hat{b}$
$\hat{a}$	2.36×10^{-8}	–	3.53×10^{-8}	–
$\hat{b}$	-1.78×10^{-6}	1.39×10^{-4}	-2.66×10^{-6}	2.08×10^{-4}

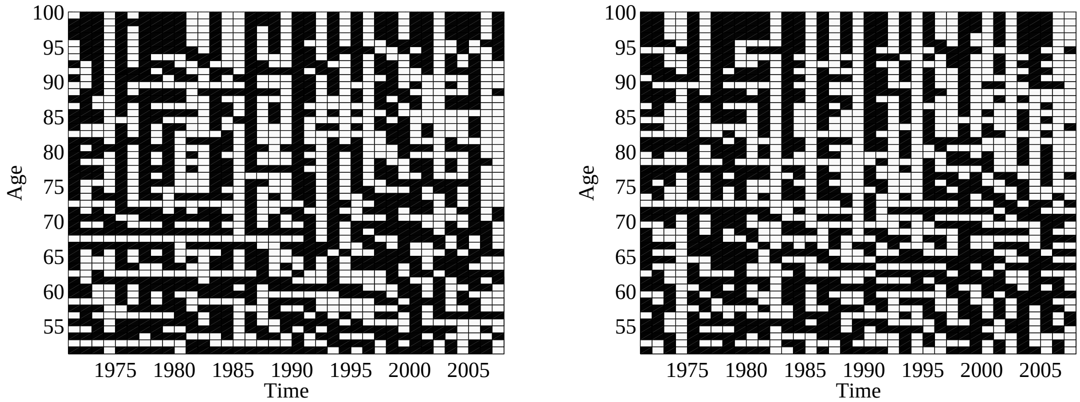


Fig. 4. Binary black-white residuals from the Wills-Sherris model for males (left) and females (right). The horizontal and vertical axes are respectively the age and the time. Black cells indicate negative residuals; white cells indicate non-negative residuals.

The estimated residuals from the Wills-Sherris model based on Australian male and female mortality data are plotted in Fig. 4. The residuals do not show distinct patterns and are consistent with the assumption of a multi-variate normal distribution. The age dependence structure is estimated as the covariance matrix of the calculated residuals.

To simulate future mortality rates, the Cholesky decomposition of the covariance matrix Σ is needed. But due to the fact that the number of years in the data is smaller than the number of ages, the Cholesky decomposition cannot be directly calculated. Instead, Wills and Sherris (2008) suggest using the eigenvalues and eigenvectors of the covariance matrix. Ω has orthogonal eigenvectors, that is:

$$VV' = I, \tag{22}$$

where V is the eigenvector matrix of Ω and I is the identity matrix. According to the property of eigenvectors, the following equation holds:

$$\Omega = V\Lambda V^{-1} = V\Lambda V' = (V\Lambda^{\frac{1}{2}})(V\Lambda^{\frac{1}{2}})', \tag{23}$$

where Λ is a diagonal matrix of Ω's eigenvalues.

Equation (23) implies that Ω can be expressed as the product of a matrix and its transpose, which is a generalised Cholesky decomposition. $V\Lambda^{\frac{1}{2}}$ can be used to simulate multi-variate normal random variables $\varepsilon(x,t)$ based on the following equation:

$$\varepsilon_t = (V\Lambda^{\frac{1}{2}})W_t. \tag{24}$$

To extrapolate mortality rates for the oldest old (101-110), we assume that the age dependence structure for these ages is the same as that for age 100. This assumption is justified by the fact that the generalised Cholesky decomposition of the age dependence matrix, $V\Lambda^{\frac{1}{2}}$, is very stable for ages above 100. Values of $V\Lambda^{\frac{1}{2}}$ for selected ages are shown in Fig. 5. The top two figures show the values of $V\Lambda^{\frac{1}{2}}$ respectively for males and females aged 60, 80, 90 and 100, suggesting that the values for these different cohorts are very different. The bottom two figures show the values of $V\Lambda^{\frac{1}{2}}$ for ages 96 to 100. The values for the oldest old are almost the same and the lines overlap.

Based on the estimates of parameters in the Wills-Sherris model and the simulated multivariate random variables, future mortality rates are projected. Male and female cohort survival probabilities derived from the projected mortality rates are shown in Fig. 6.

3.4.2 In-force probabilities

We follow Ji *et al.* (2012) and Cho *et al.* (2013) in modelling the different triggers for reverse mortgage termination, but allow mortality rates to be random variables. The mortality rate of borrowers is assumed to be lower than that of the population of the same age, in order to reflect the better health of retirees that still live at home compared to those that have moved to aged care facilities. At-home mortality rates are derived by applying age-specific scaling factors to the population mortality rates (Cho *et al.*, 2013; Ji *et al.*, 2012). The probability

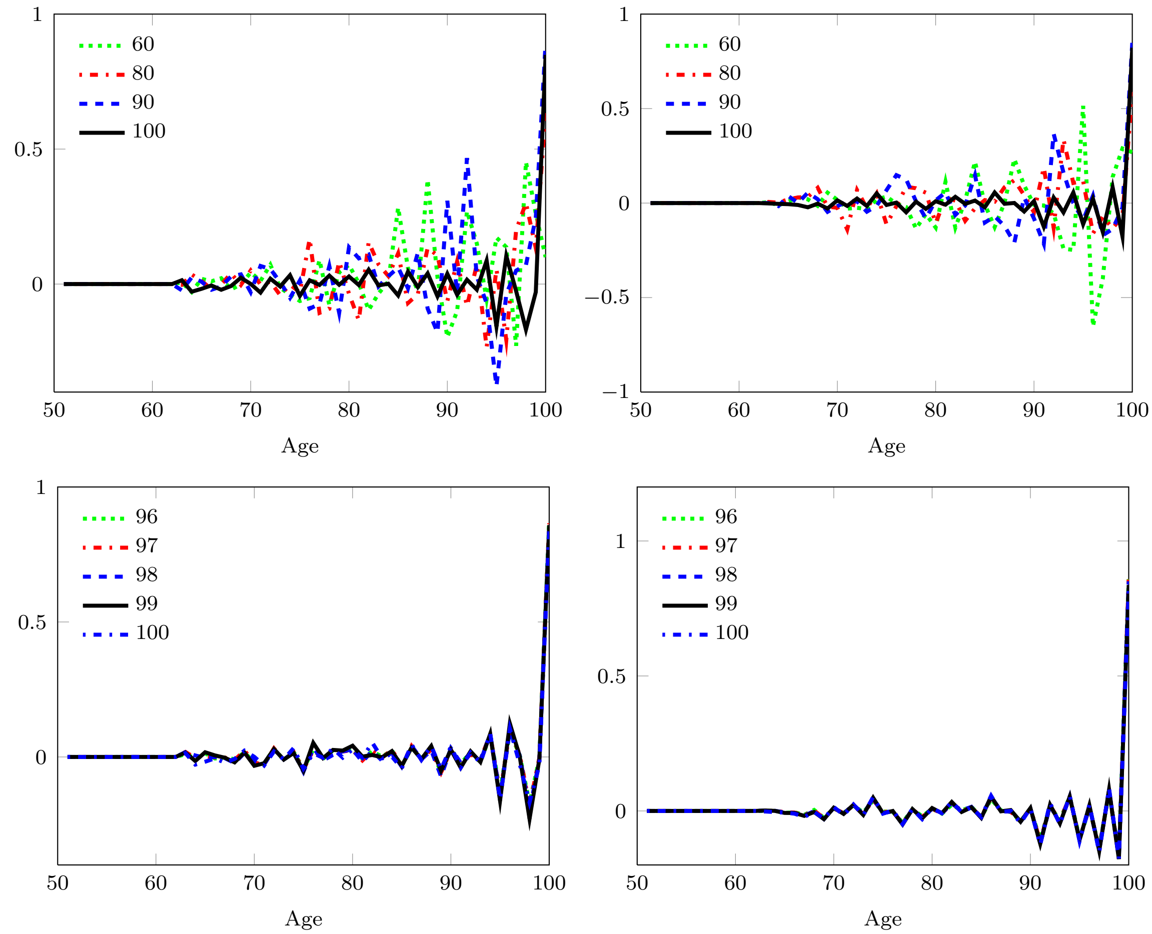


Fig. 5. The generalised Cholesky decomposition of the age dependence matrix for selected ages, males (left) and females (right).

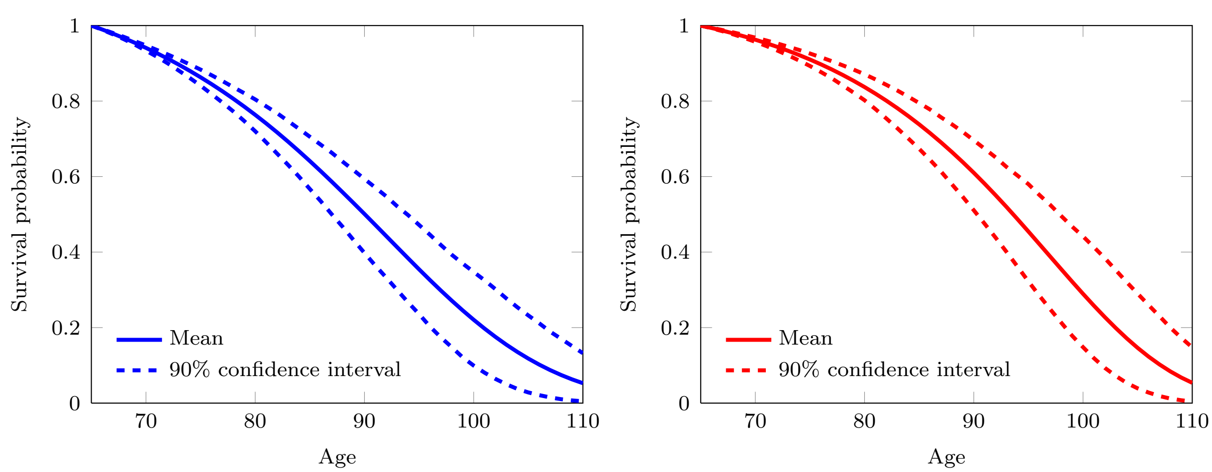


Fig. 6. Simulated survival probabilities of males (left) and females (right) initially aged 65.

of moving out due to health-related reasons, mainly because of entry into a long-term care (LTC) facility, is assumed to be a proportion of the mortality rate. Furthermore, voluntary prepayment and refinancing are specified as functions of the contract duration in years. These two termination triggers are assumed to be competing risk factors with mortality and LTC incidence.

Thus, the probability that the reverse mortgage contract is still in force at time t is a function of the mortality rate and of other termination factors:

$$ {}_tp_x^c = \exp\left\{-\int_0^t (\theta_{x+s} + \rho_{x+s})\hat{\mu}_{x+s} ds\right\} \prod_{i=1}^{t} \left[(1 - q_i^{pre})(1 - q_i^{ref})\right]^{1/4}, \tag{25} $$

where θ_{x+s} is the scaling factor for at-home mortality rates at age $x + s$, ρ_{x+s} is the age-specific factor that captures LTC incidence, $\hat{\mu}_{x+s}$ is the projected quarterly force of mortality derived from the Wills-Sherris model, q_i^{pre} is the annual duration-dependent probability of prepayment, and q_i^{ref} is the annual duration-dependent probability of refinancing. Due to lack of public access to detailed data on these rates or probabilities, assumptions based on the UK experience given in Institute of Actuaries UK (2005), Hosty *et al.* (2008), Ji *et al.* (2012) and Cho *et al.* (2013) have been employed in this paper. The parameter assumptions are summarised in Table 7. Furthermore, Equation (25) implies an assumption that the force of termination is constant within one year. Mortality rates are projected on an annual basis in Section 3.4.1. Quarterly mortality rates are obtained by assuming a constant force of mortality between integer ages.

4 Results

We present results calculated based on the projected house price indices and mortality rates to show the impact of idiosyncratic house price risk and of longevity risk on the pricing of reverse mortgages. Robustness tests are performed to test the sensitivity of the results to

Table 7. Assumptions on termination triggers adopted from Ji *et al.* (2012) and Cho *et al.* (2013).

Age	At-home mortality scaling factor	LTC incidence factor	Prepayment		Refinancing	
			Duration	Probability	Duration	Probability
65-70	0.950	0.100	1-2	0.00%	1-2	1.00%
75	0.925	0.150	3	0.15%	3	2.00%
80	0.900	0.200	4-5	0.30%	4-5	2.50%
85	0.875	0.265	6+	0.75%	6-8	2.00%
90	0.850	0.330			9-10	1.00%
95	0.825	0.395			11-20	0.50%
100+	0.800	0.460			21+	0.25%

different mortality models and to the assumptions about non-mortality termination rates.

4.1 Base case results

We model reverse mortgage loans issued to a single 65-year-old female borrower with a property valued at $800,000 at the issuance of the loan. $800,000 is about the 2010 median house price value in the data set we analyse. We assume that the mortgage rate has a quarterly lending margin of 0.4% following Chen *et al.* (2010): $\kappa = 0.4\%$ in Equation (1). The transaction cost of selling the property are assumed to be 6 % of the house price: $c = 6\%$.

In the base case, future mortality rates are projected based on the Wills-Sherris model. Different house price indices for properties with specific characteristics are compared to assess the impact of idiosyncratic house price risk. Panel A of Table 8 reports the results for a case when the house value is modelled using the aggregate house price index for Sydney. Panels B to G show the results for reverse mortgages on houses in the different regions of Sydney described in Section 3.3.3. Panels H and I illustrate the impact of the property's number of bathrooms or bedrooms. Shao *et al.* (2013) have identified these variables as important determinants of differences in house price dynamics. We also compare three different initial loan-to-value (LTV) ratios (0.2, 0.4 and 0.6) in each panel.

Table 8 reports the annualised value for the mortgage insurance premium rate π, the value of the NNEG together with the corresponding standard error, and the 95% Tail Value-at-Risk (TVaR) of the provider's shortfall. The mortgage insurance premium rate π charged for the no-negative equity guarantee is calculated by equating the values for NNEG and MIP. Based on the value for π, the provider's actual shortfall SF is calculated allowing for uncertainty in the survival probability as described in Section 3.1. The TVaR of the provider's shortfall is calculated to show the impact of stochastic mortality rates.

Mortgage insurance premium rates and NNEG values vary substantially across Panels A to I, which shows that location and house characteristics are important factors in impacting the risk of reverse mortgages. Using market-average house price dynamics substantially underestimates the risks for reverse mortgages with LTV ratios of 0.2 and 0.4 written on properties in specific regions of Sydney or with specific characteristics. For these LTV ratios, the mortgage insurance premium, the NNEG value and the TVaR are all higher in Panels B to I than the corresponding values in Panel A.

The comparison gives different results for an LTV ratio of 0.6: in this case, the NNEG value in Panel A, where the Sydney index is assumed, is higher than the NNEG in most other Panels. This can be explained as follows. At a LTV ratio of 0.6, the loan balance is very likely to exceed the house price at termination. The expected loss for the provider in that case is larger when the aggregate index is used because the aggregate index has a lower growth rate and a lower volatility than most of the disaggregated indices (see Fig. 2).

These comparisons show that reverse mortgage providers should model the house price risk in reverse mortgages using house price models that are disaggregated according to the property's location and characteristics. We illustrate this point with the following example. Suppose a reverse mortgage provider issues contracts to several 65-year-old female borrowers with different houses represented in Panels B to G of Table 8. We assume the number of properties is the same in each category. Each loan should be charged the corresponding mortgage insurance premium given in Table 8. The average annual mortgage insurance premium for this portfolio is 0.41%. This value (and each individual premium rate) is higher than the annual mortgage insurance premium rate of 0.19% calculated based on the Sydney aggregate index. In this example, pricing based on the Sydney aggregate index substantially undervalues the no-negative equity guarantee.

4.2 Sensitivity analysis: deterministic mortality

To assess the impact of longevity risk on reverse mortgage pricing we compare the results obtained using alternative assumptions on the development of future mortality rates. We first consider a simple deterministic mortality model in which future mortality rates are assumed to decrease at age-specific constant rates. The model is given by:

$$\Delta \ln \mu(x,t) = \overline{\Delta \ln \mu_x}, \tag{26}$$

where $\Delta \ln \mu(x,t) = \ln \mu(x,t) - \ln \mu(x,t-1)$ denote the year-to-year change in the log mortality rate at age x and $\overline{\Delta \ln \mu_x}$ is the sample mean of the historical changes in the log mortality rates. We estimate this model using mortality data for ages 50-100. Data on mortality rates for the oldest old are scanty and the changes in mortality rates are very volatile. We assume that mortality rates for individuals aged 101 - 110 remain constant at the rates in 2009. The assumed age-specific annual decreases in log mortality rates are shown in Fig. 7. The implied survival curve derived from the deterministic model is compared with that derived from the Wills-Sherris model in Fig. 7. The deterministic model does not account for uncertainty in survival trends and substantially underestimates future mortality improvements compared to the average projection of the Wills-Sherris model.

The first three columns of Table 8 give the mortgage insurance premium rate, NNEG and TVaR values when the deterministic mortality model is adopted. The values for LTV ratios of 0.2 and 0.4 are mostly smaller than those based on the Wills-Sherris model, suggesting

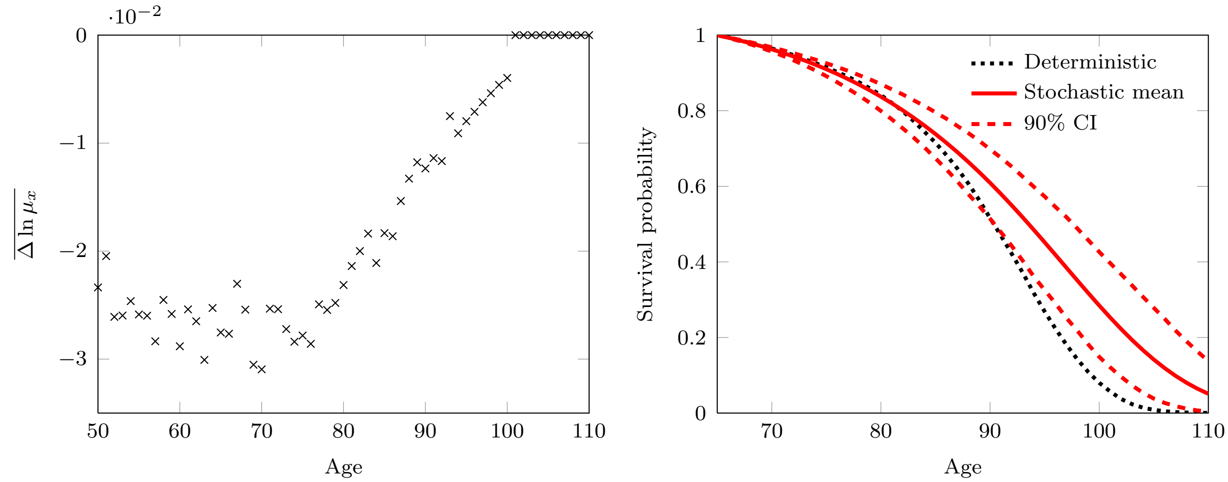


Fig. 7. Average changes in log mortality rates over time and a comparison of the survival probabilities for a 65-year-old female in the deterministic mortality model and in the Wills-Sherris model.

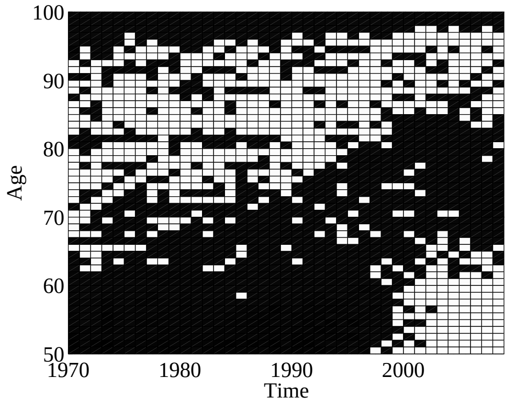
that the risk is underestimated when the provider fails to employ an appropriate mortality model to quantify and forecast mortality improvements.

The main impact of longevity risk on the pricing of reverse mortgages results from the assumed trend in mortality improvements rather than from the uncertainty around the trend. This can be seen by comparing the $\text{TVaR}_{0.95}$ values of the lender's shortfall under the deterministic mortality model and the Wills-Sherris model. The $\text{TVaR}_{0.95}$ values are small compared to the NNEG and relatively similar under both models. In addition, the impact of longevity risk is smaller than the effect of including idiosyncratic house price risk. A possible reason is the assumption that longevity risk is not priced in the market and not included in the stochastic discount factors derived from the VAR model.

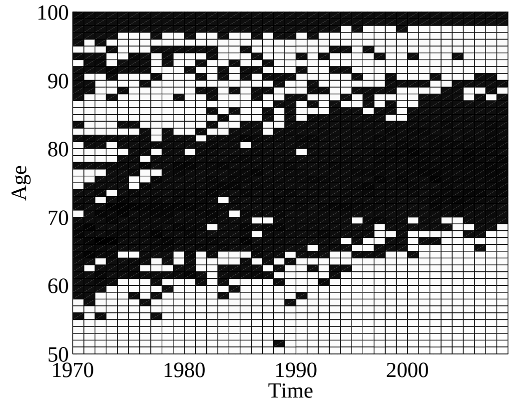
4.3 Sensitivity analysis: the two-factor Cairns-Blake-Dowd (CBD) model

To further test the results' sensitivity to the mortality assumptions we consider the popular two-factor stochastic mortality model developed by Cairns *et al.* (2006). The model is given by:

$$\text{logit } q(t, x) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}), \tag{27}$$



(a) Male



(b) Female

Fig. 8. Binary black-white residuals from the CBD model. The horizontal and vertical axes are respectively the age and the time. Black cells indicate negative residuals; white cells indicate non-negative residuals.

where $q(t,x)$ is the death probability of a person aged x at time t, $\bar{x}$ is the average age in the population, and $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ capture the period effect.

The residuals from the CBD model estimated for Australian males and females (ages 50-100) are plotted in Fig. 8. The figures show pronounce clustering of residuals from the CBD model. A possible reason can be the fact that the actual age effect shows more curvature than the logit-linear specification in the two-factor CBD model.

The residuals from the Wills-Sherris model and from the two-factor CBD model are compared. We average the age-specific residuals over time and calculate their standard deviations. The resulting values are shown in Fig. 9. The residuals from the two-factor CBD model are generally smaller and much less volatile than the residuals from the Wills-Sherris model but show patterns in Fig. 8. This reflects the different model assumptions for cohort, period and age trends as well as the different number of factors for volatility and assumptions for dependence between cohorts.

We also compare the projected survival curve for a 65-year-old female based on the CBD model with that based on the Wills-Sherris model. The survival curves are shown in Fig. 10.

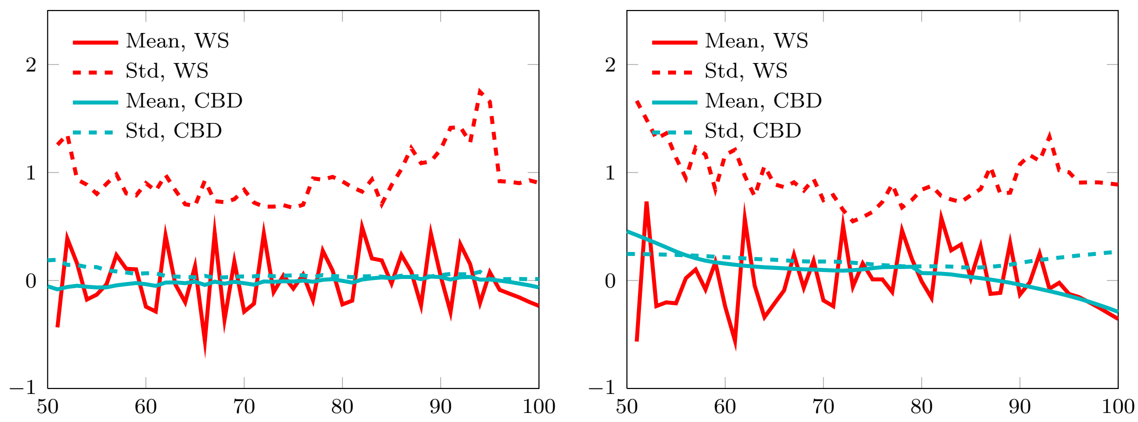


Fig. 9. Comparison of residuals from the Wills-Sherris model and the two-factor CBD model, males (left) and females (right).

The survival probabilities projected using the CBD model are much lower than those estimated from the Wills-Sherris model and very similar to those calculated from the deterministic mortality model. The uncertainty around the average survival curves is very comparable in the two stochastic mortality models.

The last three columns of Table 8 show the mortgage insurance premium rates, NNEG and TVaR values based on the CBD model. All values are very close to those calculated based on the deterministic model. The values are generally less than those calculated under the Wills-Sherris model for low LTV ratios (0.2 and 0.4) and greater for high LTV ratios (0.6). These differences are explained by the different longevity trends projected in the Wills-Sherris model. In situations where the accumulated loan amount exceeds the house value (more likely for contracts with high LTV ratios), a longer life expectancy increases the chance that the house price catches up. This effect is comparable to the price of an in-the-money put option: the longer the time to maturity, the lower the price of an in-the-money option and the higher the price of an out-of-the-money option.

Table 8. Valuation of the mortgage insurance premium rate π and the NNEG for reverse mortgages with different loan-to-value (LTV) ratios.

Model	Deterministic			Wills-Sherris			Cairns-Blake-Dowd		
LTV	0.2	0.4	0.6	0.2	0.4	0.6	0.2	0.4	0.6
A. Overall Sydney house price index									
π (p.a.)	0.003%	0.230%	3.246%	0.009%	0.360%	2.583%	0.003%	0.237%	3.126%
NNEG	71	12,794	400,017	279	22,393	335,952	90	13,147	379,366
S.E.	17	498	2,131	36	639	2,038	18	491	2,094
TVaR	0.000	0.000	0.000	0.467	6.048	12.913	0.179	5.278	13.487
B. Price index for houses near the central business district									
π (p.a.)	0.218%	0.720%	1.829%	0.239%	0.711%	1.621%	0.218%	0.716%	1.819%
NNEG	6,043	42,421	186,092	7,298	46,370	181,302	6,036	42,138	184,776
S.E.	470	1,673	4,092	494	1,680	3,876	463	1,651	4,048
TVaR	0.000	0.000	0.000	6.654	17.148	29.594	6.424	17.779	31.168
C. Price index for houses near to coastlines									
π (p.a.)	0.076%	0.255%	1.184%	0.088%	0.302%	1.183%	0.076%	0.257%	1.173%
NNEG	2,062	14,238	110,932	2,624	18,645	124,031	2,070	14,284	109,598
S.E.	289	879	2,399	308	939	2,402	286	866	2,359
TVaR	0.000	0.000	0.000	4.387	11.923	21.331	3.893	11.512	22.120
D. Price index for houses near to an airport									
π (p.a.)	0.243%	0.492%	0.967%	0.247%	0.484%	0.901%	0.242%	0.491%	0.966%
NNEG	6,748	28,189	88,181	7,570	30,584	90,594	6,735	28,142	87,983
S.E.	565	1,552	3,146	572	1,554	3,087	558	1,538	3,123
TVaR	0.000	0.000	0.000	8.041	19.035	31.435	8.063	19.653	32.754
E. Price index for houses near to a park									
π (p.a.)	0.111%	0.494%	2.720%	0.134%	0.596%	2.267%	0.112%	0.495%	2.662%
NNEG	3,049	28,339	311,635	4,051	38,270	280,287	3,076	28,376	302,862
S.E.	350	1,129	3,250	374	1,205	3,059	345	1,110	3,181
TVaR	0.000	0.000	0.000	5.391	13.544	21.830	4.951	13.445	22.989
F. Price index for houses near to harbour									
π (p.a.)	0.146%	0.506%	1.754%	0.173%	0.579%	1.652%	0.146%	0.506%	1.729%
NNEG	4,007	29,090	176,722	5,228	37,101	185,682	4,024	29,034	173,539
S.E.	394	1,218	3,075	414	1,285	3,013	386	1,195	3,014
TVaR	0.000	0.000	0.000	6.119	14.260	23.117	5.858	14.269	24.042
G. Price index for all houses excluding B - F									
π (p.a.)	0.040%	0.377%	3.392%	0.058%	0.519%	2.649%	0.041%	0.381%	3.291%
NNEG	1,079	21,307	426,785	1,721	32,963	348,165	1,116	21,539	408,843
S.E.	190	801	2,762	211	913	2,574	188	787	2,697
TVaR	0.000	0.000	0.000	2.766	9.418	16.412	2.167	8.928	16.983
H. Price index for houses with less than or equal to two bathrooms									
π (p.a.)	0.010%	0.247%	3.078%	0.019%	0.374%	2.485%	0.011%	0.253%	2.968%
NNEG	269	13,752	370,431	561	23,275	318,003	294	14,080	352,317
S.E.	86	566	2,239	99	692	2,148	87	558	2,198
TVaR	0.000	0.000	0.000	1.028	6.913	13.748	0.634	6.170	14.350
I. Price index for houses with more than two bathrooms									
π (p.a.)	0.058%	0.418%	2.868%	0.081%	0.540%	2.376%	0.059%	0.420%	2.781%
NNEG	1,577	23,759	335,272	2,412	34,438	298,788	1,612	23,871	321,653
S.E.	209	893	2,871	232	1,005	2,717	205	874	2,807
TVaR	0.000	0.000	0.000	3.391	10.145	17.505	2.914	9.912	18.420

S.E. is the standard error of the NNEG value. TVaR is the Tail Value-at-Risk of the lender's shortfall at the significance level of 95%. 'Deterministic', 'Wills-Sherris' and 'Cairns-Blake-Dowd' denote different mortality models.

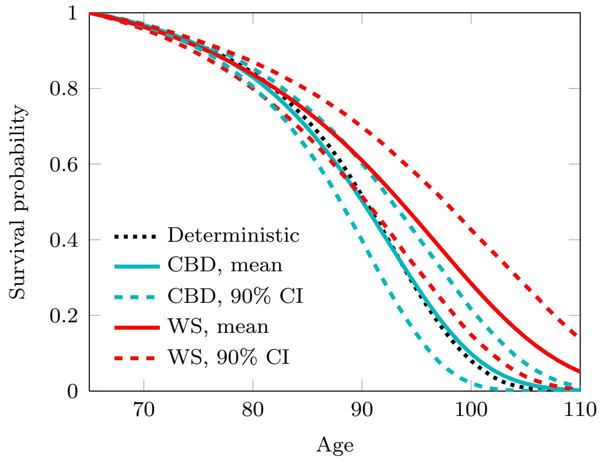


Fig. 10. Comparison of survival probabilities of the cohort 65 from the Wills-Sherris model and the two-factor CBD model.

4.4 Sensitivity analysis: LTC incidence, prepayment and refinancing

Other termination triggers such as move-out due to health related reasons, voluntary prepayment and refinancing are also important risk factors faced by reverse mortgage providers. In the base case analysis we use assumptions on these rates and probabilities shown in Table 7. This section tests the sensitivity of the base case results by varying the assumptions on the LTC incidence, prepayment probabilities and refinancing probabilities. The results are shown in Table 9.

The numerical results show that the annual mortgage insurance premium rates are stable for different assumptions on these termination rates and probabilities. Even in a joint stress test where the LTC incidence, prepayment probabilities and refinancing probabilities are decreased or increased by 50% at the same time, annual mortgage insurance premium rates show limited variations. The values of the NNEG and the TVaR are also stable in the different scenarios. Based on the results shown in Tables 8 and 9, we conclude that the impact of idiosyncratic house price risk and longevity risk is much larger than that of non-mortality termination triggers like LTC incidence, prepayment and refinancing.

Table 9. Sensitivity analysis: valuation of the mortgage insurance premium π and the NNEG for reverse mortgages for alternative assumptions about LTC incidence, prepayment and refinancing probabilities.

	Base	LTC Incidence		Prepayment		Refinancing		Joint	
		↓ 50%	↑ 50%	↓ 50%	↑ 50%	↓ 50%	↑ 50%	↓ 50%	↑ 50%
A. Overall Sydney house price index									
π (p.a.)	0.360%	0.377%	0.337%	0.388%	0.335%	0.384%	0.338%	0.430%	0.293%
NNEG	22,393	24,232	20,315	25,189	19,945	25,974	19,310	31,566	15,665
S.E.	639	657	611	698	585	727	561	814	492
TVaR	6.048	6.067	5.945	6.485	5.642	6.811	5.366	7.290	4.914
B. Houses near the central business district									
π (p.a.)	0.711%	0.690%	0.719%	0.719%	0.701%	0.728%	0.692%	0.710%	0.688%
NNEG	46,370	46,433	45,593	48,933	43,952	51,721	41,545	54,367	38,674
S.E.	1,680	1,666	1,673	1,753	1,611	1,855	1,521	1,914	1,453
TVaR	17.148	16.877	17.253	17.625	16.703	18.694	15.732	18.835	15.398
C. Houses near to coastlines									
π (p.a.)	0.302%	0.304%	0.296%	0.314%	0.291%	0.315%	0.290%	0.327%	0.272%
NNEG	18,645	19,363	17,727	20,175	17,245	21,076	16,485	23,638	14,517
S.E.	939	946	922	994	888	1,046	842	1,115	784
TVaR	11.923	11.901	11.818	12.540	11.340	13.236	10.733	13.882	10.119
D. Houses near to an airport									
π (p.a.)	0.484%	0.471%	0.490%	0.485%	0.482%	0.490%	0.477%	0.475%	0.480%
NNEG	30,584	30,682	30,144	31,914	29,323	33,612	27,821	35,087	26,291
S.E.	1,554	1,542	1,550	1,607	1,504	1,697	1,423	1,740	1,376
TVaR	19.035	18.779	19.133	19.476	18.627	20.627	17.573	20.746	17.282
E. Houses near to a park									
π (p.a.)	0.596%	0.596%	0.583%	0.625%	0.568%	0.627%	0.565%	0.652%	0.526%
NNEG	38,270	39,580	36,289	41,942	34,964	43,894	33,365	49,497	28,943
S.E.	1,205	1,208	1,187	1,280	1,137	1,349	1,076	1,433	1,001
TVaR	13.544	13.419	13.530	14.132	12.991	14.979	12.241	15.458	11.720
F. Houses near to harbour									
π (p.a.)	0.579%	0.578%	0.569%	0.603%	0.556%	0.606%	0.553%	0.625%	0.519%
NNEG	37,101	38,295	35,347	40,351	34,141	42,258	32,559	47,240	28,569
S.E.	1,285	1,288	1,265	1,365	1,210	1,440	1,146	1,530	1,063
TVaR	14.260	14.099	14.248	14.926	13.622	15.821	12.844	16.306	12.247
G. All houses excluding B - F									
π (p.a.)	0.519%	0.530%	0.496%	0.554%	0.486%	0.552%	0.487%	0.597%	0.435%
NNEG	32,963	34,840	30,535	36,809	29,573	38,235	28,428	44,928	23,692
S.E.	913	924	890	983	850	1,032	808	1,120	734
TVaR	9.418	9.390	9.335	9.957	8.906	10.514	8.429	11.047	7.892
H. Houses with less than or equal to two bathrooms									
π (p.a.)	0.374%	0.389%	0.352%	0.401%	0.348%	0.398%	0.351%	0.441%	0.307%
NNEG	23,275	25,048	21,234	26,087	20,804	26,946	20,107	32,439	16,458
S.E.	692	709	666	751	639	785	610	870	543
TVaR	6.913	6.945	6.797	7.373	6.482	7.758	6.156	8.274	5.670
I. Houses with more than two bathrooms									
π (p.a.)	0.540%	0.549%	0.521%	0.574%	0.509%	0.573%	0.509%	0.612%	0.461%
NNEG	34,438	36,185	32,138	38,214	31,075	39,776	29,816	46,166	25,157
S.E.	1,005	1,015	981	1,083	934	1,137	888	1,232	805
TVaR	10.145	10.073	10.106	10.723	9.606	11.337	9.075	11.846	8.544

S.E. is the standard error of the NNEG value. TVaR is the Tail Value-at-Risk of the lender's shortfall at the significance level of 95%. The LTV ratio is 0.4 and mortality is forecasted based on the Wills-Sherris model.

5 Conclusions

This paper addresses the pricing and risk analysis of reverse mortgages allowing for idiosyncratic house price risk and longevity risk. The impact of idiosyncratic house price risk and longevity risk are shown to be large.

To model idiosyncratic house price risk, disaggregated house price indices are constructed using the hybrid hedonic-repeat-sales house price model developed in Shao *et al.* (2013). A VAR(2) model is employed to generate economic scenarios that include projections of a city-level house price index. Based on the VAR model stochastic discount factors that reflect the macroeconomic risks impacting reverse mortgage cash flows are calculated. Disaggregated house price indices are projected using a VARX(1,0) model with the aggregate house price index as the exogenous variable. The Wills-Sherris stochastic mortality model is calibrated and employed to forecast future mortality rates. Other termination triggers, including move-out due to health related reasons, voluntary prepayment and refinancing, are linked to the projected stochastic mortality rates.

We find that pricing reverse mortgages based on an average house price index substantially underestimates the risks underwritten by the provider for low loan-to-value ratios of 0.2 and 0.4. Failing to accurately incorporate the cohort trend of improvements in mortality rates also underestimates the risk for low LTV ratios. Opposite effects are found for a high LTV ratio of 0.6. These results agree with the findings of Alai *et al.* (2013), who find that reverse mortgages with LTV ratios of over 50% have different risk profiles than contracts with lower loan to value ratios.

Our results are also in line with other studies that focus on analysing the impact of longevity risk on reverse mortgage pricing and risk management. Li *et al.* (2010) compare NNEG values using period life tables for 2007 and a cohort life table derived from the Lee-Carter model. They find that NNEG values are typically larger when cohort life tables are used, but the differences are not statistically significant. Wang *et al.* (2008) and Yang (2011) analyse the securitization of longevity risk in reverse mortgages. Wang *et al.* (2008) focus on longevity bonds for reverse mortgages. They test the sensitivity of the present value of the bond values to different mortality assumptions and find that the impact of mortality shocks is very limited. Yang (2011) develops "collateralised reverse mortgage obligations". She compares the fair spreads for different tranches using the two-factor CBD model, the Lee-Carter model and a static mortality table. She finds that assuming a static mortality table overestimates the fair spread for all tranches, with differences of up to 30% for the senior tranche.

Our results suggest that risk factors associated with a property's characteristics and a stochastic mortality model based on cohort trends should be used in the pricing of reverse mortgage loans. The study provides new and improved insight into the design of reliable and affordable home equity release products.

Acknowledgement

The authors acknowledge the financial support of the Australian Research Council Centre of Excellence in Population Ageing Research (project number CE110001029). Shao also acknowledges the financial support from the Australian School of Business and the China Scholarship Council. Opinions and errors are solely those of the authors and not of the institutions providing funding for this study or with which the authors are affiliated.

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