CESifo Working Papers

www.cesifo.org/wp

Tenancy Rent Control and Credible Commitment in Maintenance

Richard Arnott
Elizabeta Shevyakhova

CESIFO WORKING PAPER NO. 4608
CATEGORY 12: EMPIRICAL AND THEORETICAL METHODS
JANUARY 2014

CESifo *Working Paper No. 4608*

Tenancy Rent Control and Credible Commitment in Maintenance

Abstract

Under tenancy rent control, rents are regulated within a tenancy but not between tenancies. This paper investigates the effects of tenancy rent control on housing quality and maintenance. Since the discounted revenue received over a fixed-duration tenancy depends only on the starting rent, intuitively the landlord has an incentive to spruce up the unit between tenancies in order to "show" it well, but little incentive to maintain the unit well during the tenancy. The paper formalizes this intuition and presents numerical examples illustrating the efficiency loss from this effect.

JEL-Code: R210, R310, R380.

Keywords: tenancy rent control, rent control, maintenance, housing quality, credible commitment.

*Richard Arnott**
Department of Economics
University of California, Riverside
USA – Riverside CA 92521
richard.arnott@ucr.edu

Elizaveta Shevyakhova
Powerlytics, Inc.
Doylestown / Philadelophia / USA
liza.shevyakhova@gmail.com

*corresponding author

January 7, 2014
The authors would like to thank seminar participants at the MIT Real Estate Seminar and the Strategy and Business Economics Division Seminar at the UBC Sauder School, and session participants at the 2005 North American Regional Science Association Meetings, especially the discussant, John Quigley, and at the Lincoln Institute Conference, Present and Retrospect: The Work of John M. Quigley, for useful comments. Arnott would like to thank Emmanuel Garcia for expert research assistance. This paper is dedicated to the memory of John Quigley. As much as anyone, John forged the character of modern urban economics, with its healthy interplay between theoretical and empirical analysis, and its concern for practical public policy issues, particularly those related to the disadvantaged. John's energy did much to vitalize the field, and his generosity to junior colleagues (including Arnott) and graduate students was legend. This paper is a doubly appropriate tribute, because John's principal research interest was housing economics, and his critical comments on an earlier version of the paper provided the spur for the "serious" calibration of section 5's numerical example.

1. Introduction

Tenancy rent control is a form of rent control in which rents are regulated within a tenancy but may be raised without restriction between tenancies; more specifically, the starting rent for a tenancy is unregulated but the path of nominal rents within a tenancy, conditional on the starting rent, is regulated, typically causing rents to rise less rapidly over the tenancy than they would in the absence of controls[1]. Many, perhaps most, jurisdictions around the world that previously had traditional first- and second-generation rent control programs (Arnott, 1995) have moved in the direction of tenancy rent control as a method of partial decontrol[2].

In jurisdictions that have stricter forms of rent control, tenancy rent control may be an attractive method of partial decontrol. Because the starting rent adjusts to clear the market, tenancy rent control does not generate the excess demand phenomena (such as key money, waiting lists, and discrimination) of stricter rent control programs, and should have a less adverse effect on the matching of households to housing units[3]. Tenancy rent control continues to provide sitting tenants with improved security of tenure; for one thing, rent regulation within tenancies precludes economic eviction; for another, because tenancy rent control, like other forms of rent control, provides landlords with an incentive to evict tenants, it is invariably accompanied by conversion (rehabilitation, demolition and reconstruction, and conversion to condominium) restrictions[4]. As well, tenancy rent control may be a politically attractive method of partial decontrol since it continues to provide rent protection to sitting tenants, who are typically the strongest opponents of decontrol. These benefits must be weighed against the costs. The most obvious costs are the tenant lock-in created by tenancy rent control and the unfairness of the preferential treatment of sitting tenants. There are also less obvious costs. The workability of

[1] This defines the "ideal type", which is what will be modeled in this paper. Many jurisdictions have forms of rent control that are intermediate between tenancy rent control, according to the above definition, and more traditional forms of rent control. In some, rent increases are regulated both within and between tenancies, but less severely between tenancies than within tenancies. In others, rent increases are unregulated between tenancies but are subject to a variety of regulatory provisions within a tenancy, such as a guideline rent increase (which allows rents to rise by a certain percentage per year) with a cost-pass-through provision (which allows the landlord to apply for a rent increase above the guideline rent increase if justified by cost increases).

[2] Basu and Emerson (2000, 2003) and Arnott (2003) list some of these jurisdictions. Borsch-Supan (1996) models the current German system and Iwata (2002) the current Japanese system, both of which are termed "tenant protection" systems.

[3] There is a large literature on the adverse effects of rent control. Three particularly good papers that avoid polemical rent-control bashing are Frankena (1975), Glaeser and Luttmer (2003), and Olsen (1988).

[4] Miron and Cullingworth (1983) examine the effects of rent control on security of tenure.

tenancy rent control makes it more difficult to move to complete decontrol, should this be deemed desirable. Also, because a rent control administration is kept in place, it is relatively easy to return to harder controls should the political winds change. Landlords, fearing this, may curtail investment[5].

This paper focuses on another less obvious cost of tenancy rent control – its adverse effect on maintenance. Pollakowski (1999) provides an empirical analysis of the effects of New York City's rent control system on housing maintenance there. Arnott and Johnston (1981) provides an informal, diagrammatic discussion of the effects of several rent control programs (though not tenancy rent control) on housing quality and maintenance. This paper will adapt the model of Arnott, Davidson, and Pines (1983) to examine how the application of tenancy rent control to a single atomistic landlord-builder affects his profit-maximizing behavior[6].

Assume, as we will throughout most of the paper in order to abstract from the tenant lock-in effect, that tenancy duration is exogenous. There are two conflicting intuitions concerning the effects of tenancy rent control on the atomistic landlord's behavior. A lay person with good economic intuition would probably argue that tenancy rent control gives the landlord an incentive to spruce up his units between tenancies so that they "show" well and hence can be let at a higher starting rent, but little incentive to maintain the units well during tenancies, since, after the starting rent has been agreed upon, maintaining well has no effect on the rent stream during the tenancy. An economist might however reasonably object that, with tenancy duration exogenous, there is nothing to prevent the landlord from following the program that is profit-maximizing in the absence of tenancy rent control — which we shall term the *efficient program.* If the landlord follows this program, the tenant should be willing to pay as much over her tenancy as she would have for an uncontrolled unit. This line of reasoning suggests that, were it not for the tenancy lock-in, the landlord's profit-maximizing program would be unaffected by the application of tenancy rent control.

---

[5] These less obvious costs are evident in the Ontario experience with rent control (e.g., Smith, 2003).

[6] Since the analysis is "very" partial equilibrium, it will ignore the effects of tenancy rent control on the level of rents and on other markets such as the labor market.
While the paper focuses on tenancy rent control, the techniques employed can be applied to examine the effects of other forms of rent control on the landlord's optimal program (indeed, Arnott and Johnston (1981) does so, albeit informally).

The resolution of the two conflicting intuitions lies in the ability of the landlord to credibly commit to the efficient program. If he is able to credibly commit to a maintenance program, he will credibly commit to the efficient program and the tenant will agree to pay the same in rent in discounted terms over the duration of the tenancy as in the absence of rent control. The landlord will therefore be making the same revenue and incurring the same costs as in the absence of rent control, and can surely do no better than this. Thus, if the landlord can credibly commit to the efficient program, tenancy rent control alters the timing of rent payments over a tenancy but generates no inefficiency.

If, however, the landlord is unable to credibly commit to pursuing the efficient program, once the lease is signed he has an incentive to pursue a different maintenance program, which we term the *opportunistic program*. Since the signing of the lease fixes the discounted rent the landlord will receive over the current tenancy, the only incentive he has to maintain is to improve the quality of the unit at the end of the lease, as this will increase the discounted rent he receives on subsequent tenancies. Compared to the efficient program, the opportunistic program entails both a reduction in average maintenance and a postponement of maintenance within a tenancy. Before the lease is signed, a prospective tenant should in this situation realize that under tenancy rent control the landlord will pursue the opportunistic rather than the efficient maintenance program and hence not be willing to pay as high a starting rent as she would if he were to pursue the efficient program.

The crux of the matter is therefore the landlord's ability, under tenancy rent control, to commit to a particular maintenance program. Three commitment mechanisms might be partially effective. The first is contracting on maintenance. One problem with this commitment mechanism is that, since maintenance is such an amorphous concept, maintenance clauses in the lease would be highly incomplete; for example, if the contract were to require the landlord to replace appliances every ten years, he might replace with appliances that are used and reconditioned or of minimal quality. Another problem is that it would be costly for a tenant to document sufficiently well to meet the standard of evidence of the courts and real estate tribunals that her landlord had not met the maintenance terms of the contract. This is the familiar problem of costly state verification. The second commitment mechanism, reputation, is likely to be ineffective since the typical prospective tenant knows little or nothing about different landlords' maintenance performance when she is searching for a unit. The third mechanism, maintenance regulation, suffers from problems similar to those for contracting on maintenance. In our judgment, such commitment devices are largely ineffective, and in our analysis we shall assume them to be completely ineffective. The efficiency costs that we identify are reduced to the extent that these commitment mechanisms are indeed effective.

Section 2 presents a preliminary, stripped-down model that highlights the maintenance distortion caused by tenancy rent control when landlords are unable to credibly commit to the optimal program, which we term the commitment-in-maintenance (contractual) failure. Section 3 presents the central model in the absence of rent control, which is a particular case of Arnott, Davidson, and Pines (1983). Section 4 applies the central model to the analysis of tenancy rent control. Section 5 presents a calibrated example focusing on the magnitude of the efficiency loss caused by tenancy rent control. Section 6 briefly discusses how the paper's modeling of the housing market might be extended to provide a richer treatment of tenancy rent control, and briefly notes some additional effects of tenancy rent control this richer treatment leads to. Section 7 concludes.

2. A Stripped-Down Version of the Model

The central model is quite complex, employing optimal control theory. To elucidate the economics, we start with a stripped-down model. The model considers the profit-maximizing maintenance choices, in a stationary environment, of a landlord who buys a unit of housing (i.e., a unit area of floor space) and then rents it out to the same tenant[7] for two equal-length periods, at the end of which the tenant moves out and the landlord sells the unit. A unit's quality in a period, q_t, is a function of its quality over the previous period, q_{t-1}, and maintenance expenditure undertaken at the end of the previous period, m_{t-1}[8]:

$$q_t = g(q_{t-1}, m_{t-1}), \tag{1}$$

[7] The analysis is conducted per unit of housing (i.e., per unit area of floor space).
[8] Appendix C collects all the notation used in the paper.

which we term the quality change function. In the absence of rent control, in each of the two periods the tenant pays the uncontrolled, market-determined rent as a function of its quality in that period. The quality of a unit is measured by its market-determined rent, and maintenance exhibits positive and diminishing marginal returns[9]. With tenancy rent control, the landlord and tenant negotiate a starting rent, which is nominally the rent for the first period of the tenancy, with the second-period rent being some administratively determined function of the first-period rent, and with maintenance expenditures during the tenancy being non-contractible. To further simplify, it is assumed here and throughout the paper that rent-controlled units constitute a sufficiently small fraction of rental housing units that they do not affect the uncontrolled rent function[10], and that each rent-controlled unit is decontrolled at the end of the tenancy, so that it sells at price equal to the market-determined value as a function of quality, $V(q)$, for a unit of its terminal quality.

Subsection 2.1 examines the landlord's profit-maximizing maintenance decision without rent control, subsection 2.2 the corresponding problem with tenancy rent control, and subsection 2.3 presents a numerical example.

2.1. No rent control

Without rent control, the landlord's profit-maximization problem is a familiar two-period optimal investment problem with maintenance and scrap value. The present value of profit over the two periods is

$$\Pi = q_1 + \frac{q_2}{1+r} - m_0 - \frac{m_1}{1+r} - V(q_0) + \frac{V(q_2)}{(1+r)^2} \quad . \tag{2}$$

Discounted profit equals discounted rent (paid at the beginning of each period) minus discounted maintenance expenditures (incurred at the end of the period), minus the original sales price, and plus the discounted terminal sales price. The landlord maximizes (2) subject to (1) with respect to m_0 and m_1. The economics are transparent when (1) is substituted into (2) yielding

[9] Specifically, $g_m > 0$, $g_m(q_{t-1}, 0)$ equals infinity (to avoid having to deal with corner solutions), and $g_{mm} < 0$.

[10] The assumption is made with the US in mind. The typical metropolitan area there contains many jurisdictions, and, where state law permits rent control, typically each jurisdiction is free to choose whether to impose controls, as well as the form of controls. For example, rent control programs are in place in Santa Monica, Beverly Hills, Malibu, and West Hollywood, but not in any other cities in Los Angeles County.

$$\Pi = g(q_0, m_0) + \frac{g(g(q_0,m_0),m_1)}{1+r}$$

$$- m_0 - \frac{m_1}{1+r} - V(q_0) + \frac{V\big(g(g(q_0,m_0),m_1)\big)}{(1+r)^2} \quad . \tag{3}$$

The first-order conditions for m_0 and m_1 are

$$m_0: \ \frac{\partial q_1}{\partial m_0}\left[1 + \left(\frac{\partial q_2}{\partial q_1}\right)\left(\frac{1}{1+r} + \frac{V'(q_2)}{(1+r)^2}\right)\right] - 1 = 0 \tag{4}$$

$$m_1: \ \left(\frac{\partial q_2}{\partial m_1}\left(1 + \frac{V'(q_2)}{1+r}\right) - 1\right)\frac{1}{1+r} = 0\,, \tag{5}$$

where a ' denotes the derivative of a single-variable function and the partial derivatives are for the corresponding quality change function. Spending a dollar more on maintenance at the end of period 0 increases the present value of revenue in three ways. First-period quality increases by $\frac{\partial q_1}{\partial m_0}$. This increases first-period rent by the same amount, and second-period quality by $\frac{\partial q_2}{\partial q_1}\frac{\partial q_1}{\partial m_0}$ which increases discounted second-period rent by this amount divided by $1 + r$, and the discounted terminal sales price by this amount times $V'(q_2)$ divided by $(1 + r)^2$. Eq. (5) has an analogous interpretation.

2.2. Tenancy rent control

The landlord signs a lease with the tenant at the end of period zero, for the two periods, before he has chosen m_0. The lease specifies the starting rent, q_1, for period 1, and an administrative formula determines rent for period 2 as a function of the starting rent. Since the landlord cannot commit to his expenditures on m_0 and m_1, and since he decides on them after signing the lease and therefore after the discounted rent payable by the tenant has been determined, he chooses maintenance expenditure to maximize the discounted sales price minus discounted expenditures on maintenance, which we refer to as Z:

$$Z = -m_0 - \frac{m_1}{1+r} + \frac{V\big(g(g(q_0,m_0),m_1)\big)}{(1+r)^2} \quad . \tag{6}$$

The first-order conditions for m_0 and m_1 are

$$m_0: \ -1 + \left(\frac{\partial q_2}{\partial q_1}\frac{\partial q_1}{\partial m_0}\right)\frac{V'(q_2)}{(1+r)^2} = 0, \tag{7}$$

$$m_1: \; -\frac{1}{1+r} + \frac{\partial q_2}{\partial m_1}\frac{V'(q_2)}{(1+r)^2} = 0. \tag{8}$$

Eq. (7) states that the landlord spends on period-zero maintenance up to the point where the last dollar increases the discounted sales price by one dollar. Spending more on maintenance does not increase the rent received in either the first or second period. The interpretation of (8) is analogous.

Comparing (4) and (7), it appears that first-period maintenance is lower under tenancy rent control since, under rent control, spending more on maintenance at the end of period 0 does not increase either first- or second-period rent. The marginal benefit of first-period maintenance is therefore lower with rent control than without, while the marginal cost is the same. Comparing (5) and (8), it appears that second-period maintenance is lower under rent control too. Thus, the application of tenancy rent control appears to discourage maintenance. We say "appears to" since the reasoning is based on a comparison of two single equations, each from a different system of equations[11].

Comparing the pair of equations (4) and (7) with the pair of equations (5) and (8), it appears as well that tenancy rent control causes a postponement of maintenance during the tenancy. The application of rent control causes the marginal benefit of maintenance to shift down proportionally more for period-zero compared to period-one maintenance, since the effect of maintenance on discounted rent relative to its effect on sales price is larger in the first than in the second period.

The effects of tenancy rent control are more complicated than the above model suggests. First, contrary to one of the model assumptions, tenancy rent control is not typically applied to just a single tenancy but rather to a succession of tenancies. If in this situation tenancy rent control is applied to only a small fraction of the rental housing market, so that the demand for controlled housing at each quality level is perfectly elastic, then the deadweight loss associated with

[11] Consider the maximand
$\theta\left(g(q_0,m_0) + g(g(q_0,m_0),m_1)\frac{1}{1+r}\right) - m_0 - \frac{m_1}{1+r} - V(q_0) + V\big(g(g(q_0,m_0),m_1)\big)\frac{1}{(1+r)^2}$. With $\theta = 1$, this is the maximand in the absence of rent control, and with $\theta = 0$, this is the maximand under tenancy rent control (except for the constant, $-V(q_0)$, that does not affect the first-order conditions). The two programs can be rigorously compared by totally differentiating the first-order conditions with respect to θ.

tenancy rent control is fully capitalized into the sale price function for controlled housing. Thus, one should distinguish between the value functions for uncontrolled and controlled rental housing. Second, if tenancy rent control is applied to more than a small fraction of rental housing or to all rental housing, determination of the effects of the controls on the rent and value functions requires a full general equilibrium analysis of the housing market. Third, the model assumes that the duration of a tenancy is fixed. But one of the effects of tenancy rent control is to increase tenancy duration. The paper's central model takes into account the endogeneity of the controlled sales price function, but maintains the assumptions that tenancy rent control is applied to only a small fraction of rental housing and that tenancy duration is exogenous.

2.3. Comparison using specific functional forms

The aim of this subsection is to provide a specific example, in which, when the landlord is unable to commit to a maintenance program, the application of tenancy rent control causes maintenance to be reduced and also postponed within a tenancy, and to explore some of the consequences. Throughout the paper quality, an ordinal concept, is cardinalized by rent, as is done in the concept of housing services. A unit of housing is taken to be a square foot of floor area, and the unit of time a year. The quality change function is given by

$$q_t = g(q_{t-1}, m_{t-1}) = (1-\delta)q_{t-1} + 2a\sqrt{m_{t-1}}\,; \tag{9}$$

in the absence of maintenance, housing depreciates at the geometric rate δ and there are diminishing returns to maintenance. It is shown later, in section 5, which develops an example for the central model with the same functional forms as employed here, that the market value function of uncontrolled housing is linear in quality, $V(q) = S + I(q)$ where $I(q) = wq$. Define $\hat{V}(q)$ to be the market value function of a rent-controlled housing at the beginning of a tenancy. It is also shown in section 5 that this value function has the same slope as the uncontrolled market value function but differs in the constant term.

Consider first the situation without rent control. Substituting $\frac{\partial q_2}{\partial q_1} = 1-\delta$, $\frac{\partial q_1}{\partial m_0} = \frac{a}{\sqrt{m_0}}$, $\frac{\partial q_2}{\partial m_1} = \frac{a}{\sqrt{m_1}}$, and $V' = w$ into (4) and solving yields

$$m_0 = a^2\left(w\frac{1-\delta}{(1+r)^2} + 1 + \frac{1-\delta}{1+r}\right)^2 \quad (10)$$

$$m_1 = a^2\left(\frac{w}{1+r} + 1\right)^2. \quad (11)$$

With rent control the corresponding expressions are

$$\hat{m}_0 = a^2\left(w\frac{1-\delta}{(1+r)^2}\right)^2$$

$$\hat{m}_1 = a^2\left(\frac{w}{1+r}\right)^2, \quad (12)$$

where a hat over a variable indicates its value with tenancy rent control. Because of the functional forms chosen, both with and without rent control the levels of maintenance in both periods are independent of quality. It is evident that $m_0 > \hat{m}_0$ and $m_1 > \hat{m}_1$; rent control causes a cutback in maintenance in both periods. Also,

$$\frac{\hat{m}_0}{\hat{m}_1} = \left(\frac{1-\delta}{1+r}\right)^2 < \frac{m_0}{m_1} = \left(\frac{1-\delta}{1+r} + \frac{1+r}{w+1+r}\right)^2,$$

indicating that rent control causes maintenance to be cut back proportionally more at the end of period 0 than at the end of period 1, and in this sense to be postponed.

We now turn to a numerical example. The unit of time is a year, of distance is a foot, and of money is a dollar. We consider a housing unit with initial quality of $q_0 = 25.0$ ($/ft^2-year), which falls in the quality range of apartments in the extended example of section 5. That section calibrates the central model to accord with empirical regularities, obtaining $r = 0.0678$, $\delta = 0.2034$, and $a^2 = 0.4905$. With the cardinalization of quality and the quality change function that we have assumed, the value function has a constant slope[12] of $(1 - \delta)(1 + r)/(r + \delta) \equiv w = 3.136$. Applying these parameters to the stripped-down model gives $m_0 = 7.601$ ($/ft^2-yr), $m_1 = 9.724$, $\hat{m}_0 = 2.354$, and $\hat{m}_1 = 4.230$.

---

[12]With the assumed quality change function, value can be decomposed into *intrinsic value*, which is what the unit would be worth if nothing were spent on maintenance and is directly proportional to quality, and *surplus from maintenance*, which is a constant, independent of quality. We denote intrinsic value by $I(q)$. Letting w denote its slope, since $I(q_0) = q_1 + I(q_1)/(1 + r)$ and since $q_1 = (1 - \delta)q_0$, $w = (1 - \delta)(1 + r)/(r + \delta)$.

In the absence of rent control, applying (9) gives that $q_1 = 23.776$ and $q_2 = 23.307$. Since competition between landlords drives profit to zero, from (2),

$$V(q_0) = q_1 + \frac{q_2}{1+r} - m_0 - \frac{m_1}{1+r} + \frac{V(q_2)}{(1+r)^2} .$$

Substituting $V(q_2) = V(q_0) - w(q_0 - q_2)$ into this equation yields

$$V(q_0) = \left(q_1 + \frac{q_2}{1+r} - m_0 - \frac{m_1}{1+r} - \frac{w(q_0 - q_2)}{(1+r)^2}\right) / \left(1 - \frac{1}{(1+r)^2}\right) ,$$

which equals 197.098, and then $V(q_2) = 191.789$. To put these numbers in context, consider a 1000 ft^2 apartment. The apartment has a value today of $197,098. The landlord immediately spends $7,601 in maintenance and receives the year's rent of $23,776. A year from today, he spends $9,724 in maintenance and receives the year's rent of $23,307. Two years from today he sells the apartment for $191,789.

With tenancy rent control, applying (9) again yields $\hat{q}_1 = 22.063$ and $\hat{q}_2 = 20.456$. Also, recognizing that controls apply only for the two-period tenancy gives $V(\hat{q}_2) = V(q_0) - w(q_0 - \hat{q}_2) = 182.847$. On the assumption that the present value of rent over the controlled tenancy is what it would be in an uncontrolled unit providing these first- and second-period qualities, the value of the controlled apartment today is

$$\hat{V}(q_0) = \hat{q}_1 + \frac{\hat{q}_2}{1+r} - \hat{m}_0 - \frac{\hat{m}_1}{1+r} + \frac{V(\hat{q}_2)}{(1+r)^2},$$

which equals 195.265. Thus, applying tenancy rent control over the single, two-year tenancy results in a discounted deadweight loss for a 1000 ft^2 apartment of about $1,832. This number may appear small, but we shall show in the extended example of section 5 that the deadweight loss is considerably larger when tenancies are longer and when controls are applied over a longer period of time.

The above model is incomplete. We could close the model by introducing housing construction, and then derive the time path of housing quality with and without rent control, but have chosen to do so only in the context of the central model, to which we now turn.

3. The Central Model without Rent Control

The central model differs from the stripped-down model in treating time as continuous. This allows optimal control theory and phase plane analysis to be applied, and permits a neat and transparent closing of the model.

A competitive landlord-builder owns a vacant lot on which only a single unit of housing can be constructed[13]. Housing is durable and its quality is endogenous. Multiple quality-changing technologies are in principle available, including construction, maintenance, demolition and reconstruction, and rehabilitation. The quality of a unit of housing is measured by (cardinalized by) its rent. The economic environment is stationary, and is described by the quality-changing technologies and the interest rate. The quality-changing technologies include the construction cost function relating construction cost to quality, and the maintenance technology, which is autonomous – the unit's rate of quality change depends on its current quality and the current level of maintenance expenditure but not on the unit's age *per se*. The landlord chooses the profit-maximizing program. Under these assumptions, phase plane analysis shall be employed.

A general analysis of "the landlord's problem" is presented in Arnott, Davidson, and Pines (1983). In contrast to that paper, we modify the specification by cardinalizing quality in terms of rent rather than by construction cost. We focus on the practically most realistic case in which, in the absence of controls, at the beginning of the program it is profit maximizing to construct[14] and then to downgrade to saddle-point quality[15]. In an earlier version of the paper (Arnott and Shevyakhova, 2007), the quality-changing technologies permitted three qualitatively different active programs: initial construction followed by downgrading to saddle-point quality, a construction-downgrading-demolition cycle, and initial construction followed by a downgrading-rehabilitation cycle. We showed there, as intuition would suggest, that which program is profit-maximizing depends on the relative costs of maintenance, construction, and rehabilitation. Here, to simplify, we restrict the analysis to the case of initial construction followed by downgrading to

[13] The analysis can be extended straightforwardly to endogenize structural density (Arnott, Davidson, and Pines, 1986).

[14] We imagine that housing is produced under constant returns to scale, that the market is growing, and that the market rent and value functions are constant over time. New housing units are continually being constructed, perhaps by adding storeys onto existing buildings, perhaps by building on vacant land at a prespecified structural density. We analyze the problem of a landlord who is about to add an extra housing unit, and who therefore decides on construction quality and the time path of maintenance on the unit. Except for the construction decision, the same analysis applies to a landlord who purchases a previously-constructed unit.

[15] Saddle-point quality is that quality at which quality does not change when the landlord applies the profit-maximizing level of maintenance.

saddle-point quality, essentially assuming that, both with and without rent control, this program is profit-maximizing, which occurs when maintenance is cheap relative to construction and rehabilitation, and is broadly consistent with one variant of the classical filtering hypothesis, which states that in an unregulated housing market, actual quality-changing technologies are such that housing deteriorates as it ages[16].

3.1. Analysis of optimal program without rent control

The quality of a unit of housing is measured by its rent. At time 0 the landlord constructs a single housing unit of quality q_0 on his lot and then downgrades the unit asymptotically to saddle-point quality q_∞. Where $q(t)$ is quality at time t, $m(t)$ maintenance expenditure at time t, r the interest rate, $C(q)$ the cost of constructing a housing unit of quality q, and $g(q,m)$ the quality change function[17], the profit-maximizing program without rent control is the solution to

$$\max_{q_0,m(t)} \int_0^\infty \big(q(t)-m(t)\big)e^{-rt}dt - C(q_0)$$

$$\text{s.t.} \quad \begin{aligned} &i)\ \dot{q} = g(q,m) \\ &ii)\ q_0 = q(0)\ free \\ &iii)\ \lim_{T\uparrow\infty} q(T)\ \ free\,. \end{aligned} \tag{13}$$

Note that tenant maintenance is not considered. We impose non-negativity conditions on q and m. Where 's denote derivatives and subscripts partial derivatives, we also impose reasonable restrictions on the functions g and C: $i)\ g_q < 0, g_m(q,0) = \infty, g(q,0) < 0, g_m(q,\infty) = 0, g_m > 0, g_{mm} < 0$; and $ii)\ C'(q) > 0, C''(q) < 0$ and $C(0) = 0$. Thus, there are positive but diminishing returns to maintenance; holding fixed the rate of quality deterioration, more has to be spent on maintenance as quality increases; and with zero maintenance, the unit deteriorates. Also, construction entails no fixed cost and exhibits positive and increasing marginal cost. In our numerical examples, the first-order conditions of the optimal program will define a unique interior maximum.

[16] There are many different variants of the filtering hypothesis. Another states that in an unregulated housing market, a housing unit is occupied by relatively poorer households as it ages. Another way of saying this is that it is efficient for the poor to live in hand-me-down housing. This variant of the filtering hypothesis has been central to economic arguments against public housing, since public housing entails the construction of new housing for the poor.

[17] Note also that $g(q,m)$ is the negative-of-depreciation function.

We solve the problem using optimal control theory (Kamien and Schwartz, 1991). The current-value Hamiltonian corresponding to (13) is

$$q(t) - m(t) + \phi(t) g\big(q(t), m(t)\big)\,, \tag{14}$$

where $\phi(t)$ is current-value co-state variable on $\dot{q} = g\ (q, m)$. The first-order condition[18] for maintenance is

$$-1 + \phi(t) g_m\big(q(t), m(t)\big) = 0\,. \tag{15}$$

Since $\phi(t)$ is the marginal value of quality at time t, and $g_m(q(t), m(t))$ the amount by which quality is increased by an extra dollar's expenditure on maintenance, ϕg_m is the marginal benefit from maintenance. Thus, at each point in time, maintenance should be such that marginal benefit equals marginal cost. The conditions imposed on g_m guarantee that there is a unique, interior optimal level of maintenance expenditure for all non-negative values of q and ϕ; thus, we may write $m = m(q, \phi)$ with $m_\phi > 0$. Inserting this function into (14) yields the maximized current-value Hamiltonian:

$$\mathcal{H}(q, \phi) = q - m(q, \phi) + \phi g\big(q, m(q, \phi)\big)\,. \tag{16}$$

The equation of motion of the co-state variable is

$$\dot{\phi} = r\phi - \mathcal{H}_q = r\phi - 1 - \phi g_q\,. \tag{17}$$

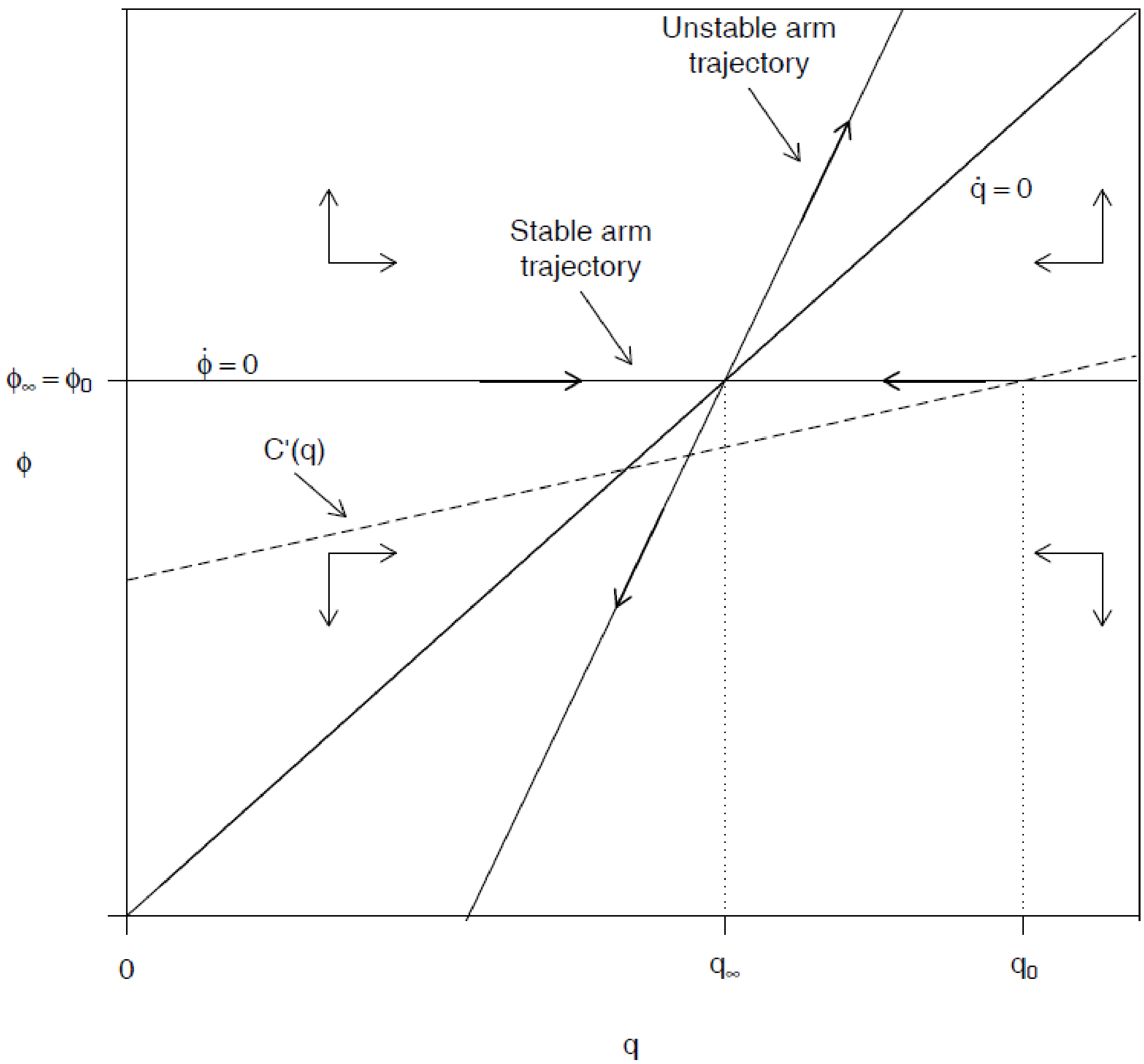
The assumptions thus far have not ruled out the possibility that the optimal saddle-point program entails upgrading to saddle-point quality via maintenance alone. We assume that the maintenance and construction technologies are such that the saddle-point program entails construction at the start of the program, which is satisfied if $\phi_\infty > C'(q_0)$. The transversality condition with respect to q_0 is then

$$\phi(0) = C'(q_0)\,. \tag{18}$$

[18] Throughout the analysis we shall omit second-order conditions but we check that they are satisfied in our numerical examples.

Construction quality should be increased up to the point where the marginal value of quality equals its marginal cost.

Figure 1: Phase plane for construction with downgrading to the steady state



We are now in a position to construct the phase plane corresponding to this program. We assume that: i) the $\dot{q} = 0$ locus is positively sloped; ii) the $\dot{\phi} = 0$ locus is either negatively sloped or less positively sloped than the $\dot{q} = 0$ locus; and iii) the $\dot{q} = 0$ locus and $\dot{\phi} = 0$ locus intersect in the positive orthant. Thus, there is a unique saddle-point, (q_∞, ϕ_∞). Figure 1 displays a phase plane consistent with these assumptions. As is the case for all subsequent figures, Figure 1 is drawn for the functional forms and parameters used in the numerical example presented in Section 5.

We also have the infinite horizon transversality conditions associated with terminal quality and terminal time. Arnott, Davidson, and Pines (1983) proved that, under the assumptions made, these conditions imply that the optimal trajectory must terminate at the saddle-point. Putting together the necessary conditions for optimality, we obtain that the optimal program entails construction at quality $q_0 = q(0)$, at which the marginal cost of construction $C'(q_0)$ is equal to the marginal value of quality $\phi_0 = \phi(0)$. After construction the quality declines along the stable arm to the saddle point.

For an autonomous optimal control problem with discounting, the value of the program at any time along an optimal trajectory equals the value of the Hamiltonian at that time divided by the interest rate:

$$V(t) = \frac{\mathcal{H}(q(t),\phi(t))}{r} \ .$$

The economic interpretation is that the value of the Hamiltonian gives the economic return per unit time from owning the property, which includes the net (of expenses and depreciation) earnings stream it generates plus capital gains, and competitive asset pricing requires that the net return per unit time from owning an asset equal the asset price times the discount rate. The value of the program immediately after initial construction is then $\mathcal{H}(q_0, \phi_0)/r$, so that the value of the program immediately before initial construction is $V^* = \mathcal{H}(q_0, \phi_0)/r - C(q_0)$.

4. Tenancy Rent Control

We model tenancy rent control as specifying a time path of rent over the duration of the tenancy, conditional on the starting rent.[19] Let q_0 denote the starting rent, t the length of time into the tenancy, $F(q_0, t)$ (with $\partial F/\partial q_0 > 0$)) the rent control function (which specifies the maximum allowable rent t years into a tenancy conditional on q_0), and $\hat{q}(t)$ the rent charged by the landlord t years into a tenancy. A tenancy rent control program imposes the constraint that $\hat{q}(t) \leq F(q_0, t)$.

[19] In fact, tenancy rent control programs typically specify a time path of *maximum* allowable rent over the duration of the tenancy, or a maximum percentage annual increase in nominal rent, over the duration of the tenancy, conditional on the starting rent. Thus, our modeling of tenancy rent control assumes that it is profit maximizing for the landlord to set the time path of rent equal to the time path of maximum rent over the duration of the tenancy.

4.1. Assumptions

We shall examine the effects of tenancy rent control applied to a single housing unit when all other units are uncontrolled; the analysis is therefore partial equilibrium. We make a number of simplifying assumptions:

Assumption 1. *The length of a tenancy is exogenous at L.*

This assumption is made for two reasons. First, we wish to abstract from the effect of tenancy rent control on tenancy duration, in order to focus on its effects on landlord maintenance and conversion. Second, the assumption takes into account that tenancy rent control is invariably accompanied by restrictions on eviction[20]. Since tenancy rent control front-end loads rent over a tenancy, shorter tenancies are more profitable for landlords. In the absence of restrictions on eviction, tenancy rent control would therefore provide landlords with an incentive to evict tenants[21].

Assumption 2. *The rent control function is such that the landlord finds it profit maximizing to charge the maximum controlled rent over the duration of a tenancy, i.e.* $\hat{q}(t) = F(q_0, t)$.

This assumption states that, under the opportunistic program, the time path of controlled rents over a tenancy are sufficiently front-end loaded relative to the time path of market rents that the tenancy rent control constraint binds strictly throughout the tenancy. While not primitive, this assumption greatly simplifies the analysis since otherwise the possibility would have to be considered that the rent control constraint binds over some quality intervals but not over others.

Assumption 3. *Tenants are identical.*

Assumption 4. *Tenants face perfect capital markets and discount financial flows at the same rate as the landlord.*

We assume that information is public and that tenants are fully rational, and so calculate the opportunistic maintenance program. The starting rent under tenancy rent control adjusts such

---

[20] We use the term eviction to mean that the tenant is required to leave her unit even though she would prefer not to, rather than in the legal sense.

[21] Tenancy rent control rules out economic eviction (raising rents to force a tenant out). But this still leaves the possibility that the landlord can evict a tenant by rehabilitating or demolishing the unit, or by citing the tenant for minor lease violations, or by leasing the unit to family members, or by converting it to owner occupancy, or by withdrawing it temporarily from the market. Assumption 1 rules out these possibilities.

that a tenant is indifferent between living in a controlled and uncontrolled unit, which occurs if and only if the discounted value of controlled rents over the tenancy equals the discounted value of market rents for the same quality path.[22] The assumption that the tenant's discount rate is the same as the landlord's is made to simplify the analysis.

Under the above assumptions, the opportunistic program is independent of the form of the rent control function. A proof runs as follows. Suppose that the profit-maximizing program with a particular rent control function has been solved for. Now modify the rent control function, holding constant the time path of maintenance, and therefore of quality, but allowing the starting rents for each tenancy to adjust so that tenants remain indifferent between controlled and uncontrolled housing. The profitability of the program remains unchanged and the landlord cannot improve profitability by altering the program. Without ambiguity, we may then let $\hat{q}(t;\ q_0)$ denote the time path of quality over a tenancy under the opportunistic program, conditional on starting quality q_0. The condition that, with the opportunistic program, over each tenancy the discounted value of controlled rents equals the discounted value of market rents may then be written a

$$\int_0^L F(q_0\,, t)e^{-rt}dt = \int_0^L \hat{q}(t; q_0)e^{-rt}dt\,.$$

Thus, under the above assumptions, it is the imposition of tenancy rent control rather than its severity[23] that matters since it is the imposition of tenancy rent control that undermines the credibility of the efficient program.

[22] This statement is strictly correct when apartment floor areas are identical. If apartment sizes differ, in an unregulated housing market and with zero moving costs, equilibrium might entail tenants in lower-quality housing choosing to live in larger apartments, for example. In contrast, since a tenancy rent control contract applies to a specific apartment-tenant pair, a tenant living in a rent-controlled apartment does not having the option of changing apartment size during a tenancy. When apartment sizes differ, there are utility functions such that the statement remains strictly correct (e.g., the utility function that is linear and monotonically increasing in the apartment floor area is in this family). But more generally our analysis fails to take into account an additional source of deadweight loss under tenancy rent control deriving from tenants' inability to adjust apartment size.

[23] A tenancy rent control program is more severe than another if it permits a lower nominal percentage increase in rent every year during a tenancy. Assumption A.2 is that the tenancy rent control program is sufficiently severe that the landlord finds it profit maximizing to charge the maximum controlled rent over the duration of the tenancy. If the tenancy rent control program is sufficiently "lax" that the landlord finds it profit maximizing to charge the maximum controlled rent over no portion of the tenancy, the program has no effect. Intermediate situations are analytically messy.

In the previous section we assumed that, in the absence of rent control, it is profit maximizing for the landlord to construct and then downgrade to saddle-point quality. In this section we assume that this is profit maximizing under tenancy rent control as well. Put alternatively, we assume that maintenance is sufficiently cheap relative to construction and rehabilitation that both with and without rent control the most profitable course of action for the landlord is to construct and then downgrade to the corresponding steady state.

4.2. Analysis of optimal program with rent control

The profit-maximizing program with rent control, too, entails construction followed by downgrading from one tenancy to the next, but maintenance follows a saw-tooth pattern, increasing within each tenancy and then falling discontinuously from the end of one tenancy to the start of the next. The program converges to a steady-state tenancy maintenance cycle in which quality is highest at the beginning and end of each tenancy, rather than to a steady-state quality.

We decompose solution of the opportunistic program under tenancy rent control during a single tenancy into two stages. In the first stage, we solve the program taking as given not only the initial quality of the unit and the duration of the tenancy but also the terminal quality. In the second stage, we solve for the profit-maximizing terminal quality. The landlord decides on this program after the lease has been signed, and therefore after his discounted rent over the tenancy has been determined. The first-stage problem entails the minimization of discounted maintenance expenditures needed to achieve terminal quality in tenancy i, $q_L^{(i)}$, taking as given the starting quality, $q_0^{(i)}$, and the tenancy duration, L. This is an elementary optimal control program with a well-known solution. Further on we slightly abuse notation by suppressing tenancy indicator i whenever it does not cause confusion.

Define $J(q_0, q_L, L)$ to be the value of this program. We shall use three properties of the solution:

$$\frac{\partial J}{\partial q_0} = \phi(0)\ , \quad \frac{\partial J}{\partial q_L} = -\phi(L)e^{-rL}\ , \ \ \dot{\phi} = r\phi - \phi g_q\ , \tag{19}$$

where $\phi(t)$ is the current value of the co-state variable on $\dot{q} = g(q, m)$ at time t. The first solution property indicates that $\phi(0)$ is the marginal value of quality at the start of the tenancy, after the tenancy contract has been signed. The second indicates that $\phi(L)$ is the marginal value of terminal quality at terminal time, so that $\phi(L)e^{-rL}$ is the marginal value of terminal quality discounted to the beginning of the tenancy. Since the first stage of the problem entails deciding on the maintenance path over the tenancy, after the contract has been signed, we refer to ϕ as the *ex post* (*viz.,* after the tenancy contract has been signed) marginal value of quality. The last solution property is that over a tenancy the *ex post* marginal value of quality grows[24] at the rate $r - g_q$ through the tenancy.

The second stage of the solution of the opportunistic program entails the choice of q_L. To derive this, we work with a value function. Under tenancy rent control, the value of a housing unit is a function not only of quality but also of how much time remains in the current tenancy contract[25]. Let $\hat{V}(q)$ denote the value of a housing unit of quality q between tenancies, and $R(q_0)$ the revenue received over a tenancy contract, discounted to the beginning of the tenancy contract. The landlord decides on the maintenance program, and hence q_L, after signing the tenancy contract, and therefore after the revenue received over the tenancy has been determined. Then the value function $\hat{V}(q)$ may be written as

$$\hat{V}(q_0) = R(q_0) + \max_{q_L}\{J(q_0, q_{L,}L) + \hat{V}(q_L)e^{-rL}\}\,. \quad (20)$$

Terminal quality is chosen to maximize the expression in curly brackets in (20). The corresponding first order condition is

$$\frac{\partial J}{\partial q_L} + \hat{V}'(q_L)e^{-rL} = 0\,. \quad (21)$$

Comparing the second equation in (19) and (21) yields

$$\phi(L) = \hat{V}'(q_L\,)\,. \quad (22)$$

[24] Suppose the landlord buys an extra unit of quality today at a price of ϕ. Instantaneously, he must make the competitive return on that unit, $r\phi$, and the return comprises two components, the capital gain minus the depreciation.

[25] Since the housing market remains competitive under rent control, it must still be the case that owning the property for an increment of time between t and $t + dt$ within a tenancy provides income of $rV(q(t), t)$ where $V(q(t), t)$ is the market value of a controlled housing unit of quality q t units of time into a tenancy. From this relationship, the rent control function, and the boundary condition that $\hat{V}(q_0) = V(q_0, 0)$, $V(q(t), t)$ may be calculated.

Differentiating (20) with respect to q_0 yields

$$\hat{V}'(q_0) = R'(q_0) + \partial J/\partial q_0 \quad \text{(using the Envelope Theorem)} \tag{23}$$

$$= R'(q_0) + \phi(0) \quad \text{(using (19))}.$$

Eq. (23) requires some care in interpretation. $\hat{V}'(q_0)$ is the *ex ante* (before the tenancy contract has been signed) marginal value of quality at the start of a tenancy, while $\phi(0)$ is the *ex post* (after the tenancy contract has been signed) marginal value of quality at the start of a tenancy. Eq. (23) indicates that, at starting quality, the *ex ante* marginal value of quality exceeds the *ex post* marginal value of quality by $R'(q_0)$, marginal discounted revenue. Thus, there is a downward jump discontinuity in the marginal value of quality at the time the lease is signed.

Now return to (22). It states that, in contrast, the marginal value of quality immediately before the termination of the tenancy equals the marginal value of quality immediately afterwards, in both cases equaling the increase in the sales price from a unit increase in terminal quality.

The value of the program immediately prior to construction is

$$\hat{V}^* = \max_{q_0^{(1)}} \left[\hat{V}\left(q_0^{(1)}\right) - C(q_0^{(1)})\right]. \tag{24}$$

Assuming an interior solution, the corresponding first-order condition for profit-maximizing construction quality is

$$\hat{V}'\left(q_0^{(1)}\right) - C'\left(q_0^{(1)}\right) = 0\,. \tag{25}$$

Comparing (23) and (25), for the first tenancy,

$$\phi_0^{(1)} = C'\left(q_0^{(1)}\right) - R'\left(q_0^{(1)}\right). \tag{26}$$

Construction occurs at that quality level for which the *ex ante* marginal value of quality via construction equals the marginal cost of quality via construction, while the *ex post* marginal value of construction quality falls short of marginal construction cost by $R'(q_0^{(1)})$.

In the steady state, quality varies within a tenancy, but the starting and terminal qualities remain constant from one tenancy to the next. Let q_σ be the optimal starting and terminal quality of a steady state cycle. Since in a steady-state tenancy $q_0 = q_L = q_\sigma$,

$$\hat{V}(q_\sigma) = \frac{1}{1-e^{-rL}}\{R(q_\sigma) + J(q_\sigma, q_\sigma, L)\}.$$

Figure 2: Phase plane for construction–downgrading to the steady-state cycle under rent control

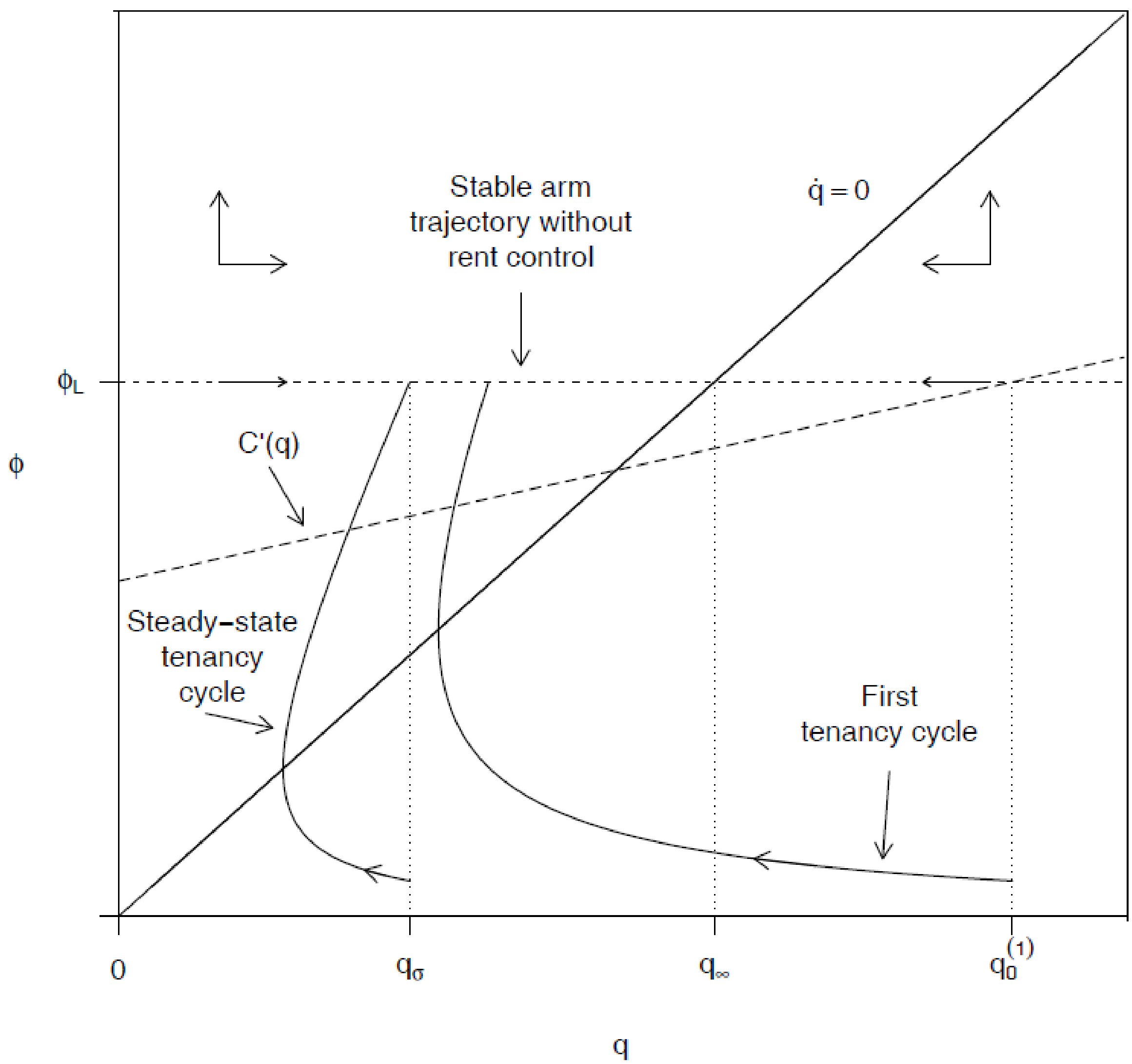


Figure 2 displays the phase diagram of the optimal program under rent control for the numerical example, and plots the optimal trajectory for two tenancies, the first tenancy that occurs immediately after construction and the steady-state tenancy. For comparison it also plots the optimal (stable arm) trajectory without rent control. With the quality change function we employ, maintenance expenditures are positively related to ϕ and independent of q. The diminished incentive to maintain under tenancy rent control is reflected in the lower position of the optimal trajectory under tenancy rent control, except at the end of each tenancy. The incentive under tenancy rent control to postpone maintenance expenditures towards the end of the tenancy is also evident.

5. Numerical Example

We had hoped to draw on the empirical literature in our choice of functional forms and parameters. Unfortunately, there appear to be no empirical studies that use Arnott, Davidson, and Pines (1983) conceptual framework as the basis of empirical analysis. As a result, we adopt the more modest goal of developing a numerical example whose parameters and functional forms are "reasonable." We chose functional forms that yield linear differential equations and closed-form value functions. In Appendix B, we obtain the parameters through calibration to the Los Angeles housing market in 2010.

As in the theoretical analysis, we measure quality by rent. We assume the following functional forms:

$$C(q) = \frac{\alpha q^2}{2} + \beta q \quad \text{and} \quad \dot{q} = -\delta q + 2a\sqrt{m} \tag{27}$$

where all the parameters are strictly positive. We assume furthermore that the parameters are such that, both with and without rent control, a steady-state program rather than a construction-demolition cycle program is optimal (i.e., profit-maximizing), in particular that it is optimal to construct and then downgrade towards the steady state. We shall parameterize the model so as to obtain a sensible optimal program in the absence of rent control.

5.1. No rent control

We start off with the situation without rent control. The current-valued Hamiltonian is

$$q(t) - m(t) + \phi(t)\left(-\delta q(t) + 2a\sqrt{m(t)}\right). \tag{28}$$

Optimal maintenance is

$$m(t) = a^2\phi(t)^2 , \tag{29}$$

so that the maximized current-valued Hamiltonian is

$$\mathcal{H} = q(t) + \phi(t)\big(a^2\phi(t) - \delta q(t)\big)\,, \tag{30}$$

and the maximized state equation is

$$\dot{q} = -\delta q(t) + 2a^2\phi(t). \tag{31}$$

The co-state equation is

$$\dot{\phi} = (r + \delta)\phi(t) - 1. \tag{32}$$

The saddle-point values of q and ϕ are

$$q_\infty = \frac{2a^2}{\delta(r+\delta)}\ , \quad \phi_\infty = \frac{1}{r+\delta}\ . \tag{33}$$

The optimal trajectory satisfies

$$\phi(t) = \frac{1}{r+\delta}\ , \quad m(t) = \frac{a^2}{(r+\delta)^2}\ ,$$

$$q(t) = \left(q(0) - \frac{2a^2}{\delta(r+\delta)}\right)e^{-\delta t} + \frac{2a^2}{\delta(r+\delta)}\ . \tag{34}$$

It is optimal to construct at that quality for which the marginal value of quality at the date of construction, $t = 0$, equals the marginal cost of quality; thus, $\phi(0) = \alpha q_0 + \beta$. Using the first equation in (34), this equation reduces to

$$q_0 = \frac{1-\beta(r+\delta)}{\alpha(r+\delta)}\ , \tag{35}$$

so that

$$q(t) = \left(\frac{1-\beta(r+\delta)}{\alpha(r+\delta)} - \frac{2a^2}{\delta(r+\delta)}\right)e^{-\delta t} + \frac{2a^2}{\delta(r+\delta)}\ . \tag{36}$$

Since $V(q) = \frac{\mathcal{H}}{r}$, the value function is

$$V(q) = \frac{q}{r+\delta} + \frac{a^2}{r(r+\delta)^2}\ . \tag{37}$$

The value function has two components. The first is what the value of the unit would be were nothing spent on maintenance, which is increasing in the unit's quality; we refer to this as the *intrinsic value,* and denote it by $I(q)$. The second component is the additional value or surplus created by maintenance, which is independent of the unit's quality; we refer to this as the *surplus from maintenance*, and denote it by S. This neat decomposability of the value function into intrinsic value and surplus from maintenance derives from the additive separability of the quality change function between quality and maintenance ($g_{qm} = 0$) when, as we have assumed, quality is cardinalized by rent. Since, as modeled, tenancy rent control operates exclusively via its effect on the maintenance program over a tenancy, independent of housing quality at the start of the tenancy, its application has no effect on intrinsic value, affecting housing value only through the surplus from maintenance. This points to a natural unitless measure of the efficiency of tenancy rent control, the *relative efficiency of maintenance* under rent control:

$$REM = \hat{S}/S \,, \tag{38}$$

where $\hat{S}$ is the surplus from maintenance with tenancy rent control and S the surplus from maintenance without rent control.

5.2. Rent control

Under rent control, the apartment is constructed at a relatively high quality, and is then downgraded in a succession of tenancy cycles, with the starting quality falling from one cycle to the next until approaching a steady-state tenancy cycle in the limit. Maintenance increases within each tenancy cycle.

The state equation is the same as without rent control, being given by (31). Because the landlord makes his maintenance decisions after the time path of rent has been determined for the duration of the tenancy, the co-state equation changes according to (19), becoming

$$\dot{\phi}(t) = (r + \delta)\phi(t). \tag{39}$$

Our solution procedure is somewhat different from that without rent control. First we solve for the optimal maintenance program within a tenancy, conditional on a constant of integration, c_0 and starting housing quality q_0. We then find the present value of maintenance and the present value of rent over a tenancy, again conditional on the constant of integration. For each tenancy, the landlord chooses the constant of integration to maximize the value of the program. Finally, we determine the starting housing quality of the first tenancy cycle by maximizing the value of the property minus construction costs with respect to $q_0^{(1)}$, thus determining the whole path $q(t)$. The details are presented in Appendix A, while here we state only the main results.

The equations for the co-state and state variables are

$$\phi(t) = c_0 e^{(r+\delta)t}, \tag{40}$$

$$q(t) = \left(q_0 - \frac{2a^2c_0}{r+2\delta}\right)e^{-\delta t} + \frac{2a^2c_0}{r+2\delta}e^{(r+\delta)t} \quad , \tag{41}$$

where c_0 and the property's quality at the beginning of a tenancy q_0 are constants to be determined. For future reference, the property's quality at the end of a tenancy cycle is

$$q_L = q_0 e^{-\delta L} + \frac{2a^2c_0}{r+2\delta}\left(e^{(r+\delta)L} - e^{-\delta L}\right). \tag{42}$$

The equation for maintenance is

$$m(t) = a^2\phi(t)^2 \tag{43}$$

In choosing the maintenance path over a tenancy with initial quality q_0, the landlord selects c_0 by maximizing $-M + \hat{V}(q_L)e^{-rL}$, where $M = \int_0^L m(t)e^{-rt}dt$ is the present value of maintenance over a tenancy cycle. The landlord's chooses

$$c_0 = \frac{1}{(r+\delta)\, e^{(r+\delta)L}}. \tag{44}$$

Having found c_0, the only unknown left is the construction quality ($q_0^{(1)}$of the very first tenancy cycle). Recall that the value without rent control equals intrinsic value plus surplus from maintenance ($V(q) = I(q) + S$). The application of rent control does not affect intrinsic value but does affect surplus from maintenance. Thus, $\hat{V}(q) = I(q) + \hat{S}$, where $\hat{S}$ is the surplus from maintenance with rent control. These results have two interesting implications. The first is that *optimal construction quality is the same with and without rent control*[26], in both cases solving the common construction quality first-order condition that marginal construction cost

[26] Since the assumed form of the quality change function is a special intermediate case in which $g_{qm} = 0$, it is natural to conjecture that, in general, the application of tenancy rent control may cause construction quality to either increase or decrease.

equal $I'(q) = \frac{1}{r+\delta}$. Thus, construction quality is determined according to (25) and is the same as without rent control in (35):

$$q_0^{(1)} = \frac{1-\beta(r+\delta)}{\alpha(r+\delta)}.$$

The second implication is that the deadweight loss due to tenancy rent control, $S - \hat{S}$, is independent of quality. The maintenance surplus under tenancy rent control is

$$\hat{S} = \frac{2KD + D^2}{4A(1 - e^{-rL})},$$

where $A = \frac{a^2}{r+2\delta}\left(e^{(r+2\delta)L} - 1\right)$; $D = \frac{2a^2(e^{\delta L} - e^{-(r+\delta)L})}{(r+2\delta)(r+\delta)}$; and $K = \frac{2a^2}{r+2\delta}\left[\frac{e^{-(r+\delta)L}-1}{r+\delta} + \frac{e^{\delta L}-1}{\delta}\right]$.

5.3. Calibration of numerical example and efficiency loss under tenancy rent control

We are now in a position to choose the parameters for the example. We calibrate on the basis of conditions in the Los Angeles metropolitan area in 2010[27], assuming that those conditions correspond to the equilibrium in the absence of rent control. We consider an apartment of average quality and measure rents and values per ft^2.

(Fact 1) Current construction costs in Los Angeles for a low- to medium-rise unit of average quality are about \$80 per ft^2 (Mewis, 2010).

(Fact 2) Operating costs for a typical apartment building, which include maintenance but exclude utilities (which are assumed to be borne by the tenant) and property taxes (which will be taken account of elsewhere), are about one-third of gross rent on average and a higher proportion on older buildings[28]. We interpret this statement to mean that, over the life of the building, the present value of rent equals three times the present value of maintenance expenditure.

(Fact 3) For the Los Angeles metropolitan area, the capitalization rate — the ratio of net operating income, defined as rent less operating expenses, including property taxes, to property value — is around 6.75% (Duffy, 2012). Since property taxes are 1.0% of the market value on

[27] This is, of course, made more difficult by the real estate meltdown in 2007/8, which resulted in, on average, a halving of residential property values and a less dramatic fall in mean rents. An offsetting advantage to calibrating on the Los Angeles area in 2010 is that housing rents and values remained more or less steady from 2008 to 2012.
[28] Conversation with Prof. Richard Peiser, Harvard University Graduate School of Design.

newer buildings, we interpret this statement to mean that, over the life of the building, the present value of rent minus the present value of maintenance expenditures is 0.0775 of the property value. Since they are a fixed proportion of market value, we incorporate property taxes in the interest rate.

(Fact 4) Rents per ft^2 vary considerably over the Los Angeles metropolitan area, but at the time we consider they averaged about \$20 per ft^2-year[29]. We take this to mean that the discounted average rent $\left(\int_0^\infty q(t)e^{-rt}dt \,/\, \int_0^\infty e^{-rt}dt = rR\right)$ equals 20.

(Fact 5) We assume that the rent on a new apartment is 50% higher than on a mature apartment.

Combined with the analytical results for the numerical example without rent control, those facts allow us to obtain values of ten unknowns (details of the calculation are presented in Appendix B): $r = 0.0678$, $\delta = 0.2034$, $a = 0.7004$, $\alpha = 0.0515$, $\beta = 2.313$, $m = \frac{20}{3}$, $q_0 = 26.67$, $q_\infty = 17.78, V_0 = 196.62$, and $V_\infty = 163.85$.

Using these parameter values, we specify the dynamics for q: from (31) $\dot{q} = -0.2034q(t) + 3.617$. Also, from (37), $I(q) = 3.687q$ and $S = 98.31$, so that $V(q) = 3.687q + 98.31$. Thus, immediately after construction at the profit-maximizing quality, one half of the property's value is its intrinsic value, with the other half being surplus from maintenance.[30]

We are agnostic concerning the reasonableness of the calibrated parameters. On one hand, the calibration was done to conform to empirical regularities for the Los Angeles housing market. On the other hand, it is the first "serious" calibration that has been performed using Arnott, Davidson, and Pines (1983) model of housing. Also, there are always issues concerning how any model, which is necessarily a simplified description of a complex reality, should be calibrated; for example, how should the calibration account for the model's ignoring uncertainty, even

---

[29] A google search of "apartment rent per square foot los angeles" on December 4, 2011 came up with numbers between \$1.50 and \$2.00 per ft^2-month.

[30] This result derives from a combination of the assumed functional form for the quality change function and assumption made in step (ii) of the calibration that $R = 3M$. Immediately after construction, property value equals the present value of rent, R, minus the present value of maintenance costs, M: $V(q_0) = R - M$. When the maintenance function takes the form $\dot{q} = -\delta q + am^\gamma$, at each point in time the ratio of the total benefit from maintenance to the expenditure on maintenance equals $1/\gamma$, so that the ratio of the surplus from maintenance to the expenditure on maintenance equals $1/\gamma - 1$. In the example, $\gamma = 1/2$, so that $S = M$. Step (ii) of the calibration entails the assumption that $R = 3M$. Combining these results with the assumption made in step (ii) of the calibration yields $V(q_0) = R - M = 3M - M = 2S$. Furthermore, since $V(q_0) = I(q_0) + S, I(q_0) = S$.

though uncertainty is no doubt empirically important, and for heterogeneity in housing type and location? In many respects, the calibration does seem reasonable; for example, an interest rate of 6.78% seems reasonable, when account is taken of risk, as does a ratio of construction cost to property value at time of construction ($C(q_0)/V(q_0)$) of about 60%. However, one calibrated parameter in particular jars with intuition. Even though the *value* depreciation rate of housing *after maintenance,* $\dot{V}/V$ (which is what the empirical literature has measured) is reasonable, achieving a maximum value of slightly over 3% immediately after construction and then declining to zero in the steady state, the calibrated *rent* depreciation rate *in the absence of maintenance*, δ, of 20% seems too high.

Under rent control, assuming a tenancy period of 10 years, $L = 10$, and using the calibrated parameter values, we obtain $\hat{S} = 44.37$. Recalling that $S = 98.31$ we have that the present value of the deadweight loss due to rent control is $DWL = 53.94$ which, recall, is per ft^2 of floor area, and that, according to (38), the relative efficiency of maintenance under tenancy rent control is 0.4513. In the numerical example, therefore, the application of tenancy rent control results in maintenance being only 45.13% as effective as in the absence of rent control.

These numbers imply that if tenancy rent control were imposed by a small city in the Los Angeles area with average rent, if the imposition of tenancy rent control had been completely unanticipated, if its imposition were to result in an average tenancy duration of ten years, and if, upon imposition, renters and landlords expected tenancy rent control to remain in effect indefinitely, then immediately upon imposition the unit would lose $\$(S - \hat{S}_0) = \$53.94$ in value per ft^2 of floor area. This number strikes us as implausibly large. There are several possible reasons. First, the assumptions listed above in deriving this number are all strong ones. Second, while the model makes the extreme assumption of prohibitively costly state verification, some aspects of landlord maintenance are verifiable, and rent control ordinances typically regulate verifiable components of maintenance. Third, the model ignored that tenants as well as landlords contribute to maintenance, and that tenants' incentive to maintain increases when landlords cut back on their maintenance. Fourth, the calibration may be flawed.

As is intuitive, the relative efficiency of maintenance under rent control falls the longer is the tenancy period. In the limit, as the tenancy period approaches zero, REM approaches 1.0, and, as the tenancy period approaches infinity, REM approaches 0.0. As is also intuitive, the relative efficiency of maintenance under rent tenancy control falls the longer the expected period of time that tenancy rent control will be applied in the future.

In the numerical example for the stripped-down model of section 2, we employed the same parameter values as here, except that the tenancy period is 2 years rather than 10 years and that tenancy rent control is applied for only a single tenancy. In that example, the discounted deadweight loss due to tenancy rent control was $1.832/ft^2. Since the discounted surplus from maintenance without rent control in that example is[31] $118.70/ft^2, the relative efficiency of maintenance is 0.9846.

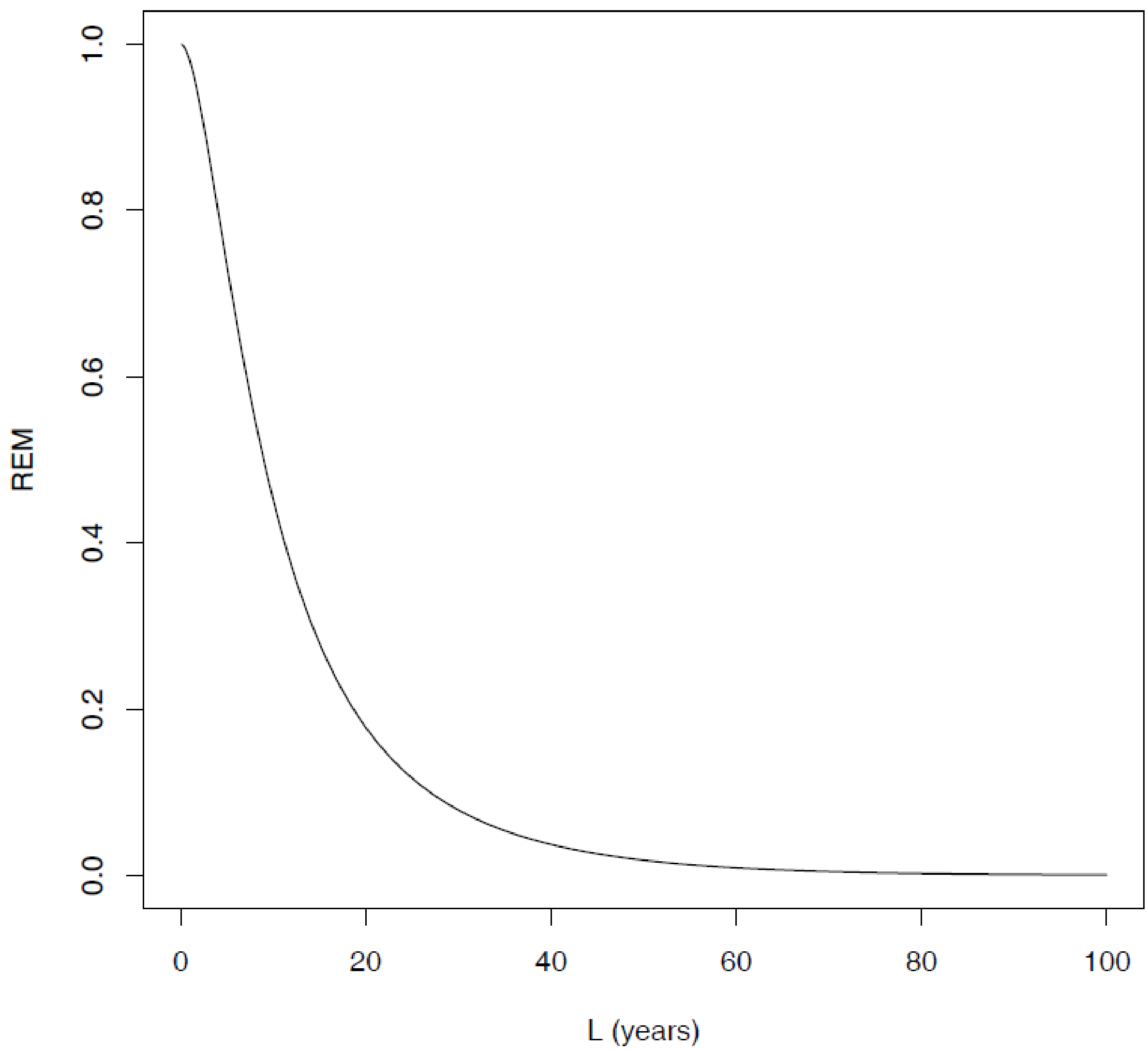
Figure 3 below plots the relative efficiency of maintenance under tenancy rent control as a function of the tenancy period, when tenancy rent control is applied forever.

In our model, we have taken the tenancy duration as fixed. In reality, however, as we have already noted, the application of tenancy rent control has a lock-in effect, causing tenancy duration to increase. The lock-in effect is stronger the more front-end-loaded are rents under tenancy rent control. There is a deadweight loss associated with the lock-in effect[32] over and above the deadweight loss identified and measured in this paper due to reduced and postponed maintenance during a tenancy.

[31] Immediately after construction with $q_0 = 25$, property value is 197.10 and intrinsic value is $I(q_0) = (1 - \delta)(1 + r)q_0/(r + \delta) = 78.39$, so that the surplus from maintenance is 118.70.

[32] Munch and Svarer (2002) examined the effects of rent control in Denmark on tenancy duration. Since the rent control system in effect at the time did not permit rents to be increased freely between tenancies, it was not tenancy rent control. They employed a cross-section analysis, exploiting differences across tenants in the benefit they derived from rent control. They found the lock-in effect to be substantial (with mean tenancy duration rising from 5.5 years for those receiving the lowest decile mean benefit from rent control to 8.7 years from those receiving the highest decile mean benefit). Unfortunately, they did not take the next step of attempting to infer the overall deadweight loss from the lock-in effect.

Figure 3: Relative Efficiency of Maintenance under Tenancy Rent Control



6. Discussion

The amount written on rent control has been quite out of proportion to its importance as a housing policy. Economists have devoted so much attention to it because rent control has provided a focal point for debating how regulated, as well as unregulated, housing markets work, and how extensive government intervention in the market should be. For example, while this paper has studied a commitment-in-maintenance failure in the context of tenancy rent control, the failure occurs more generally, in all long-term rental contracts for housing and other capital goods for which the owner does all the maintenance, whether or not the contracts are regulated.

The primary aim of the paper is to contribute to the theoretical literature on rent control in particular and on regulated housing markets in general, by introducing the commitment-in-maintenance failure and attempting to quantify its importance, *in the context of a particular model of tenancy rent control*.

Since the current version of paper is already long, we have decided not to discuss in detail how our model might be usefully extended to give a richer treatment of the housing market that would permit a more sophisticated analysis of tenancy rent control. Instead, we refer the reader to Arnott (2003), which provides a qualitative discussion along these lines, and simply mention four of the major points.

1. Our assumption that tenancy duration is fixed assumes away what is probably the most important adverse effect of tenancy rent control, the lock-in effect. There is abundant anecdotal evidence that the long-term application of rent control reduces tenant mobility, which in turn reduces labor mobility. This anecdotal evidence is supported by a limited amount of high-quality empirical work, most notably van Ommeren, Rietveld, and Nijkamp (1999), Munch and Svarer (2003), and Svarer, Rosholm, and Munch (2005). Modeling this effect in a general equilibrium context (in which the offer distribution is endogenous) is the top priority item on the tenancy rent control research agenda, and can be done by adapting the familiar search theoretical literature, for which Diamond, Mortensen, and Pissarides shared the 2010 Nobel Prize in Economics.

2. We have examined the effects of tenancy rent control on maintenance in the context of a model that takes the market rent function relating rent to housing quality, and the corresponding housing value function, as fixed. This would be appropriate if tenancy rent control were applied to a geographically small submarket within a metropolitan housing market, but then sorting effects based on tenancy duration would be important. Landlords would have an incentive either to offer lower starting rents to tenants whose observable characteristics are correlated with shorter tenancies, or, if such rent discrimination is not permitted, to choose tenants whose observable characteristics are correlated with shorter tenancies. In turn, those tenants who

anticipate the longest tenancy durations among tenants with their observable characteristics would have the strongest incentive to move into apartments under tenancy rent control. If tenancy rent control were applied to a larger housing market, a satisfactory analysis would have to solve simultaneously for the market-clearing functions relating rent and value to quality for all the relevant submarkets.

3. We considered the situation where a small submarket of housing units under tenancy rent control co-exists with a larger market of uncontrolled units. In reality, however, tenancy rent control has *always* (we know of no exception) been applied as a method of partial decontrol to housing markets that previously had stricter second-generation rent control programs with excess demand phenomena. Analysis of tenancy rent control in this more realistic situation would be considerably more difficult.

4. We assumed that the housing market is perfectly competitive. It is now well recognized, however, that housing markets are imperfectly competitive in many respects, including asymmetric information, search frictions, separation costs, incomplete insurance markets against uncertainty, and product differentiation. Taking such imperfections into account expands the range of potentially welfare-improving government intervention in housing markets. For example, since formal markets do not exist that provide tenants with insurance against rent uncertainty, tenancy rent control can be defended as a second-best method for providing such insurance. This theme is developed in Arnott (1995).

7. Conclusion

Tenancy rent control regulates rents within a tenancy, based on the starting rent, but allows the starting rent to rise without restriction between tenancies. Many jurisdictions around the world that previously had stricter first- and second-generation rent controls programs have introduced tenancy rent control as a method of partial decontrol.

This paper identified a previously unnoticed inefficiency associated with tenancy rent control, which we termed the commitment-in-maintenance failure. To simplify the argument, we supposed that tenancy durations are fixed. The signing of a unit's lease, which specifies the starting rent, determines the discounted rent over the tenancy. Since the landlord cannot increase the rent he receives over the tenancy by providing better maintenance, the application of tenancy rent control eliminates one of the incentives he has to maintain. The other incentive he has, to increase the quality of the unit at the end of the tenancy, which will increase the rental revenue he receives on subsequent tenancies, remains, and encourages maintenance towards the end of the tenancy. According to this line of argument, the application of tenancy rent control leads to the reduction and postponement of maintenance within a tenancy. A counterargument is that the landlord is free to follow the precontrol maintenance program. If the tenant believes that the landlord will do so, she should be willing to pay the same discounted rent over the tenancy as she would in the absence of controls, and if she does, tenancy rent control has no adverse effects on maintenance. The crux of the difference between the two arguments lies in the ability of a landlord to commit to a maintenance program. We argued that commitment mechanisms in this context are weak, and therefore that the application of tenancy rent control will indeed lead to the reduction and postponement of maintenance within a tenancy.

We analyzed the effects of tenancy rent control on the landlord's maintenance program using the Arnott-Davidson-Pines partial equilibrium filtering model, which regards the housing market as a continuum of quality submarkets and takes the function relating rent to housing quality as fixed.

In order to gauge the magnitude of the distortion due to the commitment-in-maintenance failure, we developed a numerical example, with parameters calibrated based on 2010 average values for the Los Angeles Metropolitan Area. Since there has been very little empirical work based on the Arnott-Davidson-Pines filtering model, we claim only that the parameter values chosen are consistent with the empirical regularities, not that they are accurate. Even if they were accurate, the numerical example would give an accurate estimate of the deadweight loss due to the commitment-in-maintenance failure only for the simplified housing market of the model, which unrealistically takes tenancy duration as exogenous, ignores uncertainty, and assumes that tenancy rent control is applied to only a small segment of the housing market. With the functional form of the quality change function assumed in the numerical example, the value of a housing unit can be separated into two components, its intrinsic value (what the unit would be worth were nothing spent on maintenance) and its (discounted) surplus from maintenance. Our central example was somewhat extreme, assuming that tenancy rent control is applied *in perpetuum* and that tenancy duration in the controlled submarket is ten years. Under these assumptions, the application of tenancy rent control results in a deadweight loss equal to somewhat over half the surplus from maintenance. The deadweight loss is smaller with a shorter tenancy duration and a shorter period of application. We interpret these results to indicate that the deadweight loss from the commitment-in-maintenance failure under tenancy rent control can be large, not that it necessarily is in practice.

Our primary intention in writing this paper was to make a positive contribution to the economic theory of rent control and more generally to housing economic theory. However, to make the paper more relevant in the context of public policy, in the course of the paper we mentioned how the model might be extended in the direction of realism and commented briefly on strengths and weaknesses of tenancy rent control as a method of partial decontrol of a second-generation rent control program.

References

Arnott, R. 1995. Time for revisionism on rent control? Journal of Economic Perspectives, 9: 99-120.

Arnott, R. 2003. Tenancy rent control. Swedish Economic Policy Review 10: 80-121.

Arnott, R., R. Davidson, and D. Pines. 1983. Housing quality, maintenance, and rehabilitation. Review of Economic Studies 50: 467-494.

Arnott, R., R. Davidson, and D. Pines. 1986. The spatial aspects of housing quality, density, and maintenance. Journal of Urban Economics 19: 190-217.

Arnott, R. and N. Johnston. 1981. Rent Control and Options for Decontrol in Ontario. Toronto, Canada: Ontario Economic Council, University of Toronto Press.

Arnott, R. and E. Shevyakhova. 2007. Tenancy rent control and credible commitment in maintenance. Boston College WP #661.

Basu, K. and M. Emerson. 2000. The economics of tenancy rent control. Economic Journal 110: 939-962.

Basu, K. and M. Emerson. 2003. Efficiency pricing, tenancy rent control, and monopolistic landlords. Economica 70: 223-232.

Borsch-Supan, A. 1996. On the West German tenants' protection legislation. Journal of Theoretical Economics 142: 380-404.

Duffy, P. 2012. Commercial real estate. Presentation at L.A. Economic Forecast Conference, Commercial Real Estate Section, June 2012. MetroIntelligence Real Estate Advisors, http://metrointel.com/LinkClick.aspx?fileticket=7jzQp6QuHzA%3d&tabid=574&mid=1260.

Frankena, M. 1975. Alternative models of rent control. Urban Studies 12: 303-308.

Glaeser, E. and E. Luttmer. 2003. The misallocation of housing under rent control. American Economic Review 93: 1027-1046.

Iwata, S. 2002. The Japanese tenant protection law and asymmetric information on tenure length. Journal of Housing Economics 11: 125-151.

Kamien, M. and N. Schwartz. 1991. Dynamic Optimization. 2nd ed. Amsterdam: Elsevier.

Mewis, R.W. (ed.). 2010. RS Means Residential Construction Cost Data 2011.

Miron, J.R. and J.B. Cullingworth. 1983. Rent control: Impact on income distribution, affordability, and security of tenure. Center for Urban and Community Studies, Toronto.

Munch, J., and M. Svarer. 2002. Rent control and tenancy duration. Journal of Urban Economics 52: 542-560.

Olsen, E. 1988. What do economists know about the effect of rent control on housing maintenance? Journal of Real Estate Finance and Economics 1: 295-307.

Pollakowski, H. 1999. Rent regulation and housing maintenance in New York City. MIT Center for Real Estate Working Paper 79.

Smith, L. 2003. Intertenancy rent control in Ontario. Canadian Public Policy 29: 213-255.

Svarer, M., M. Rosholm, and J. Munch. 2005. Rent control and unemployment duration. Journal of Public Economics 89: 2165-2181.

Van Ommeren, J., P. Rietveld, and P. Nijkamp. 1999. Job moving, residential moving, and commuting: a search perspective. Journal of Urban Economics 46: 230-253.

Appendix A: Numerical example with rent control

Here we present details on the landlord's problem under rent control. The state equation is the same as without rent control:

$$\dot{q}(t) = -\delta q(t) + 2a^2\phi(t). \tag{A1}$$

The co-state equation is now

$$\dot{\phi}(t) = (r + \delta)\phi(t). \tag{A2}$$

We start by solving an ordinary differential equation from (A2):

$$\phi(t) = c_0 e^{(r+\delta)t}, \tag{A3}$$

where c_0 is an integration constant. Substituting (A3) into the maximized state equation, (A1), gives

$$\dot{q}(t) = -\delta q(t) + 2a^2 c_0 e^{(r+\delta)t}, \tag{A4}$$

the solution of which is

$$q(t) = c_1 e^{-\delta t} + \frac{2a^2 c_0}{r+2\delta} e^{(r+\delta)t} . \tag{A5}$$

It is convenient to express one of the constants in terms of the quality at the beginning of a tenancy cycle q_0. Since

$$q_0 = c_1 + \frac{2a^2 c_0}{r+2\delta} , \tag{A6}$$

we get that

$$c_1 = q_0 - \frac{(2a^2 c_0)}{r + 2\delta}.$$

Then the quality at the end of the tenancy cycle is

$$q_L \equiv q(L) = c_1 e^{-\delta L} + \frac{2a^2 c_0 e^{(r+\delta)L}}{r + 2\delta}$$

$$= q_0 e^{-\delta L} + \frac{2a^2 c_0}{r + 2\delta}\left(e^{(r+\delta)L} - e^{-\delta L}\right). \quad \text{(A7)}$$

Recalling that $m(t) = a^2 \phi(t)^2$, we have that maintenance expenditures over a tenancy discounted to the start of the tenancy are

$$M = \int_0^L a^2\left[c_0 e^{(r+\delta)t}\right]^2 e^{-rt}\, dt = \frac{a^2 c_0^2}{r + 2\delta}\left(e^{(r+2\delta)L} - 1\right). \quad \text{(A8)}$$

Using (A5), rent over a tenancy discounted to the start of the tenancy is

$$R = \int_0^L \left(c_1 e^{-\delta t} + \frac{2a^2 c_0}{r + 2\delta} e^{(r+\delta)t}\right) e^{-rt}\, dt$$

$$= \frac{c_1}{r+\delta}\left(1 - e^{-(r+\delta)L}\right) + \frac{2a^2 c_0}{(r+2\delta)\delta}\left(e^{\delta L} - 1\right)$$

$$= \frac{1 - e^{-(r+\delta)L}}{r+\delta} q_0 + \frac{2a^2}{r+2\delta}\left[\frac{e^{-(r+\delta)L} - 1}{r+\delta} + \frac{e^{\delta L} - 1}{\delta}\right] c_0 . \quad \text{(A9)}$$

Define $A = \frac{a^2}{r+2\delta}\left(e^{(r+2\delta)L} - 1\right)$; $B = \frac{1 - e^{-(r+\delta)L}}{r+\delta}$; and $K = \frac{2a^2}{r+2\delta}\left[\frac{e^{-(r+\delta)L} - 1}{r+\delta} + \frac{e^{\delta L} - 1}{\delta}\right]$. With those constants,

$$M = A c_0^2 ,$$

$$R = B q_0 + K c_0 .$$

In choosing the maintenance path over a tenancy with initial quality q_0, the landlord maximizes $-M + \hat{V}(q_L)e^{-rL}$ by selecting c_0. Now, since $(q) = \frac{q}{r+\delta}$,

$$-M + \hat{V}(q_L)e^{-rL} = -A c_0^2 + I(q_L)e^{-rL} + \hat{S}e^{-rL}$$

$$= -A c_0^2 + \frac{q_0 e^{-(r+\delta)L}}{r+\delta} + \frac{2a^2 c_0\left(e^{\delta L} - e^{-(r+\delta)L}\right)}{(r+2\delta)(r+\delta)} + \hat{S}e^{-rL}$$

$$= -Ac_0^2 + Dc_0 + \left[\frac{q_0 e^{-(r+\delta)L}}{r+\delta} + \hat{S}e^{-rL}\right],$$

where $D = \frac{2a^2(e^{\delta L} - e^{-(r+\delta)L})}{(r+2\delta)(r+\delta)}$. Since the term in brackets is independent of the landlord's choice of maintenance path, and since the only variable in the remaining terms is c_0, the landlord's choice of maintenance path reduces to his choice of c_0. The landlord chooses

$$c_0 = \frac{D}{2A} = \frac{1}{(r+\delta)\, e^{(r+\delta)L}} \ . \tag{A10}$$

Thus,

$$M = \frac{D^2}{4A} = \frac{a^2(e^{(r+2\delta)L} - 1)}{e^{2(r+\delta)L}(r+2\delta)(r+\delta)^2} \ , \tag{A11}$$

$$R = Bq_0 + \frac{KD}{2A} = \frac{1 - e^{-(r+\delta)L}}{r+\delta} q_0 + \frac{2a^2}{(r+\delta)(r+2\delta)e^{(r+\delta)L}}\left[\frac{e^{\delta L} - 1}{\delta} + \frac{e^{-(r+\delta)L} - 1}{r+\delta}\right] . \tag{A12}$$

Finally, the construction quality is determined according to (25) and is the same as the one without rent control (35), since the intrinsic value does not change under tenancy rent control.

Recall that value equals intrinsic value plus surplus from maintenance and that the intrinsic value function is the same with and without rent control: $\hat{V}(q) = I(q) + \hat{S}$. Since $\hat{V}(q_0) = \hat{S} + I(q_0) = R - M + \hat{V}(q_L)e^{-rL} = R - M + \hat{S}e^{-rL} + I(q_L)e^{-rL}$,

$$\hat{S} = \frac{R - M + I(q_L)e^{-rL} - I(q_0)}{1 - e^{-rL}} = \frac{2KD + D^2}{4A(1 - e^{-rL})} . \tag{A13}$$

Appendix B: Calibration of the numerical example

For calibration we use (Fact 1) - (Fact 5) presented in section 5.3, together with analytical results for our numerical example without rent control. Below we list equations we solve to obtain the parameter values.

(Fact 1) together with the chosen functional form of the construction function $C(q)$ gives us

$$\frac{\alpha q_0^2}{2} + \beta q_0 = 80. \tag{B1}$$

Using the expression for $m(t)$ from (34) to calculate the present value of maintenance, we obtain

$$M = \int_0^\infty m(t)e^{-rt}dt = \frac{a^2}{r(r+\delta)^2} \ . \tag{B2}$$

The present value of rent is derived using $q(t)$ from (36):

$$R = \int_0^\infty q(t)e^{-rt}dt = \frac{1}{\alpha(r+\delta)^2} - \frac{\beta}{\alpha(r+\delta)} - \frac{2a^2}{\delta(r+\delta)^2} + \frac{2a^2}{\delta r(r+\delta)}$$

$$= \frac{1-\beta(r+\delta)}{\alpha(r+\delta)^2} + \frac{2a^2}{r(r+\delta)^2} = \frac{1-\beta(r+\delta)}{\alpha(r+\delta)^2} + 2M. \tag{B3}$$

Since we assumed that maintenance cost constitutes one third of gross rent (Fact 2), R = 3M reduces to

$$\frac{a^2}{r} = \frac{1-\beta(r+\delta)}{\alpha} \ . \tag{B4}$$

Using the expression for the value of the property (37) together with (B2) and (B3), the property value at $t = 0$ is

$$\frac{R}{r+\delta} + \frac{M}{r} = M\left[\frac{3}{r+\delta} + \frac{1}{r}\right] = M\frac{4r+\delta}{(r+\delta)r} \ . \tag{B5}$$

(Fact 2), (Fact 3), and (B5) together give us

$$R - M = 0.0775\left(M\frac{4r+\delta}{(r+\delta)r}\right)$$

or

$$\frac{(r+\delta)r}{4r+\delta} = 0.03875. \tag{B6}$$

Since $R = 3M = \frac{3a^2}{r(r+\delta)^2}$ (from (B2)), (Fact 4) gives us

$$\frac{3a^2}{(r+\delta)^2} = 20. \tag{B7}$$

The analytical results for the numerical example without rent control and (Fact 5) produce:

from (33) $\quad q_\infty = \frac{2a^2}{\delta(r+\delta)}$, (B8) from (34) $m = \frac{a^2}{(r+\delta)^2}$, (B9)

from (35) $q_0 = \left(\frac{1}{r+\delta} - \beta\right)\frac{1}{\alpha}$, (B10)

from (37) $V_\infty \equiv V(q_\infty) = \frac{q_\infty}{r+\delta} + \frac{a^2}{r(r+\delta)^2}$, (B11)

from (37) $V_0 = \frac{q_0}{r+\delta} + \frac{a^2}{r(r+\delta)^2}$, (B12)

from (Fact 5) $q_0 = 1.5q_\infty$. (B13)

Thus, we have ten equations (B1), (B4), (B6)-(B13) in the ten unknowns: r, δ, a, α, β, m, q_0, q_∞, V_0, and V_∞ . Next we solve this system.

- From (B8) and (B13), $\frac{2q_0}{3} = \frac{2a^2}{\delta(r+\delta)}$.
- Combining (B10) and (B4) gives $\frac{a^2}{r} = q_0(r + \delta)$.
- Substituting out q_0 gives $\frac{2a^2}{3r(r+\delta)} = \frac{2a^2}{\delta(r+\delta)}$, which reduces to $\delta = 3r$. Inserting this result into (B6) produces $r = 0.0678$ and $\delta = 0.2034$.
- Thus, $a^2 = \frac{20(r+\delta)^2}{3} = 0.4905$.
- From (B6) and (B7), we obtain that $m = \frac{a^2}{(r+\delta)^2} = \frac{20}{3}$.
- From (B8), $q_\infty = 17.78$, and from (B13), $q_0 = 26.67$.
- Then, from (B10), $26.67\alpha = 3.687 - \beta$, and, from (B1), $355.6\alpha + 26.67\beta = 80$. Combining these results gives $355.6\alpha + 26.67(3.687 - 26.67\alpha) = 80$ or $18.31 = 355.56\alpha$, so that $\alpha = 0.0515$ and $\beta = 2.313$.
- From (B11) and (B12) we have that $V_\infty = 163.85$ and $V_0 = 196.62$.

Appendix C: Notational Glossary

$C(q)$	cost of constructing a housing unit of quality q

$F(\cdot,\cdot)$	rent control function
$\mathcal{H}$	maximized current-value Hamiltonian
$I(q)$	intrinsic value of a housing unit
J	value of the optimal control program in which landlord minimizes discounted maintenance expenditure under rent control
L	duration of tenancy under tenancy rent control
M	the present value of maintenance expenditures (incurred over a tenancy)
R	the present value of rent (paid over a tenancy)
REM	the relative efficiency of maintenance under rent control
S	surplus from maintenance
$V(q)$	the market-determined value as a function of quality
V^*	the value of the program immediately before initial construction
$\hat{V}(q)$	value of a controlled housing unit between tenancies, as a function of quality
Z	the discounted sales price minus discounted expenditures on maintenance
a	parameter of $g()$, property's quality change function
c_0, c_1	parameters of the solution for $q(t)$, $\phi(t)$
$g(q_t, m_t)$	property's quality change function
m_t	maintenance expenditure undertaken at the end of the period t
q_L	housing quality at the end of a tenancy under tenancy rent control
q_∞	saddle-point quality in the program without tenancy rent control
q_0	housing quality at the beginning of a tenancy under tenancy rent control

$q(t)$	a unit's quality at time t
q_σ	optimal starting and terminal quality of a steady state cycle under tenancy rent control
r	discount rate
w	slope on q in $V(q)$
Π	landlord's profit
α, β	parameters of the construction function
γ	parameter of housing quality change function g()
δ	rate of quality depreciation
$\phi(t)$	marginal value of quality at time t (current-value co-state variable in the continuous time landlord's profit maximization problem)
ϕ_∞	saddle-point marginal value of quality without rent control
$\frown$	indicator of tenancy rent control